Voltage & Current Waves: Telegrapher’s equations

\[ \frac{\partial \tilde{V}}{\partial z} = -j \omega \varepsilon \tilde{I} \quad \frac{\partial \tilde{I}}{\partial z} = -j \omega \mu \tilde{V} \]

\[ \frac{\partial^2 \tilde{V}}{\partial z^2} = -j \omega \varepsilon \frac{\partial \tilde{I}}{\partial z} = -j \omega \varepsilon \left( -j \omega \mu \tilde{V} \right) \]

\[ \frac{\partial^2 \tilde{V}}{\partial z^2} + \omega^2 \varepsilon \mu \tilde{V} = 0 \quad (\tilde{V} = V^* e^{i\omega z} e^{-j\beta z} + V e^{i\omega z} e^{j\beta z} = \tilde{V}^* e^{-j\beta z} + \tilde{V} e^{j\beta z}) \]

\[ \frac{\partial^2 \tilde{I}}{\partial z^2} = -j \omega \mu \frac{\partial \tilde{V}}{\partial z} = -j \omega \mu \left( -j \omega \varepsilon \tilde{I} \right) \]

\[ \frac{\partial^2 \tilde{I}}{\partial z^2} + \omega^2 \varepsilon \mu \tilde{I} = 0 \quad (\tilde{I} = I^* e^{i\omega z} e^{-j\beta z} - I e^{i\omega z} e^{j\beta z} = \tilde{I}^* e^{-j\beta z} - \tilde{I} e^{j\beta z}) \]

These are both wave equations with \( \beta = \omega \sqrt{\frac{\mu}{\varepsilon}} \)

Although technically NOT a general result, for transmission lines this formula can typically be used since most transmission lines, if practical, ARE low-loss. The low-loss case gives this result.

Consider a voltage wave traveling in the +z direction and determine the relation between voltage and current.
This relation defines the **characteristic impedance**.

http://obiwan.cs.ndsu.nodak.edu/~ekhan/mes/programs/strip.htm

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**Lossy Transmission Lines**

Real transmission lines are lossy. First, there is loss due to the current which is modeled as a series resistance/meter.

For coaxial lines, this resistance can be found using techniques already developed.

\[
\mathcal{R} = \frac{1}{\sigma \delta} \left( \frac{1}{P_{\text{inner}}} + \frac{1}{P_{\text{outer}}} \right)
\]

The other loss is associated with the voltage between conductors and is due to dielectric loss.
This loss is modeled as a conductance/meter.

Physical basis for $G$
- Review of mechanism for dielectric current

Physical basis for dielectric loss
- Loss due to radiation
- Loss due to lattice coupling
Transmission line equations

\[
\frac{\partial \tilde{V}}{\partial z} = -(\mathcal{R} + j \omega \mathcal{L}) \tilde{I} \quad \frac{\partial \tilde{I}}{\partial z} = -(\mathcal{G} + j \omega \mathcal{C}) \tilde{V}
\]

\[
\frac{\partial^2 \tilde{V}}{\partial z^2} = -(\mathcal{R} + j \omega \mathcal{L}) \frac{\partial \tilde{I}}{\partial z} = (\mathcal{R} + j \omega \mathcal{L})(\mathcal{G} + j \omega \mathcal{C}) \tilde{V}
\]

Defining impedance/meter and admittance/meter.

\[
\tilde{Z} = \mathcal{R} + j \omega \mathcal{L} \quad \tilde{Y} = \mathcal{G} + j \omega \mathcal{C}
\]

The following form is obtained.

\[
\frac{\partial^2 \tilde{V}}{\partial z^2} = \tilde{Z} \tilde{Y} \nabla
\]

\[
\tilde{\gamma} = \sqrt{\tilde{Z} \tilde{Y}} \quad \tilde{Z}_c = \frac{\tilde{\gamma}}{\sqrt{\tilde{Y}}} = \sqrt{\frac{\mathcal{R} + j \omega \mathcal{L}}{\mathcal{G} + j \omega \mathcal{C}}}
\]

Low-loss \((\mathcal{R} \ll \omega \mathcal{L} \quad \mathcal{G} \ll \omega \mathcal{C})\)