Exercise

We will work together on the Otto Cycle Example, given last time. The answers will be posted.

Limited Pressure or Dual Cycle

Again we are assuming an ideal gas with constant ratio of specific heats.

- Some of the combustion takes place at constant volume and some takes place at constant pressure. Let $f_{cv}$ be the fraction of combustion that takes place at constant volume. Then $f_{cp} = 1 - f_{cv}$.

- Following the text, we will not worry about the residual fraction. Therefore

$$\frac{m_f}{m} = \frac{1}{1 + A/F}$$

We will know $A/F$ and $Q_{LHV}$, the fuel’s heating value.

- Lets assume we know a couple of fluid properties. Say $c_v$ and $\gamma$. We get the others: $c_p = \gamma c_v$. $R = c_p - c_v$. 
Here is how it works:

1. We start at $P_1$ and $T_1$. The specific volume is

$$v_1 = \frac{RT_1}{P_1}$$

2. We calculate the pressure and temperature at 2.

$$P_2 = P_1 r_c^\gamma$$

and

$$T_2 = T_1 r_c^{\gamma-1}$$

assuming the isentropic compression just as before. The specific volume is $v_2 = v_1 / r_c$.

3. The end of constant volume combustion is Point 3a.

$$q_{cv}^* = f_{cv} \frac{m_f}{m} Q_{LHV} = c_v (T_{3a} - T_2)$$

Since the combustion in this step was constant volume, we must have that

$$\frac{P_{3a}}{P_2} = \frac{T_{3a}}{T_2} = \alpha$$

and

$$v_{3a} = v_2$$

4. The end of constant pressure combustion is Point 3b.

$$q_{cp}^* = f_{cp} \frac{m_f}{m} Q_{LHV} = c_p (T_{3b} - T_{3a})$$

Since the combustion in this step was constant pressure,
we must have that

\[ \frac{v_{3b}}{v_{3a}} = \frac{T_{3b}}{T_{3a}} = \beta \quad \text{and} \quad p_{3a} = p_{3b} \]

5. Point 4. We again have an isentropic expansion from Point 3b to Point 4. (The piston is now back at BC.)

\[ v_4 = v_1 \quad \frac{P_{3b}}{P_4} = \left( \frac{v_4}{v_{3b}} \right)^\gamma \]

and, finally \( P_4 v_4 = R T_4 \).

Here’s a picture to help you remember.
The total heat added per kg per cycle is

\[ q^* = q_{cv}^* + q_{cv}^* \]

The work done is calculated in pieces.

\[ w_{12} = c_v (T_1 - T_2) \]
\[ w_{3a3b} = p_{3b} (v_{3b} - v_{3a}) \]
\[ w_{3b4} = c_v (T_{3b} - T_4) \]
\[ w_c = w_{12} + w_{3a3b} + w_{3b4} \]

The fuel conversion efficiency based on this gross indicated work is

\[ \eta_f = \frac{w_c}{q^*} \]

Some algebra shows that,

\[ \eta_f = 1 - \frac{1}{r_c^{\gamma-1}} \left[ \frac{\alpha \beta^\gamma - 1}{\alpha \gamma (\beta - 1) + \alpha - 1} \right] \]

This is the most general formula for fuel conversion efficiency for this type of model.
• If $\beta = 1$ the term in $[\ ]$ will be 1. This is just the constant volume combustion efficiency. The Otto cycle is a special case of the limited pressure cycle.

• If $\alpha = 1$, we have constant pressure combustion. This is the Diesel cycle. To get the efficiency, we need $\beta$. This is an additional piece of information that must be determined to completely describe this cycle.

Example Problem:

5.1

Many diesel engines can be approximated by a limited pressure cycle, a fraction of the fuel is burned at constant volume and the remaining fuel is burned at constant pressure. Suppose we have

- $\gamma = 1.3$. $c_v = 0.957$ kJ/(kg-K)
- $r_c = 15$
- $F/A = 0.045$
- Heating value of the Fuel 43,000 kJ/kg fuel.
- Inlet conditions: $P_1 = 100$ kPa, $T_1 = 289$ K.

Half the fuel is burned at constant volume and the other half at constant pressure. Draw the PV diagram and compute the fuel conversion efficiency.

Compare the efficiency and peak pressure of the cycle with that which would be obtained if the fuel was completely burned at constant pressure or at constant volume.