Modeling of System Elements

Many additional equations can be found in *Modeling and Analysis of Dynamic Systems* by Cochin and Plass.

**Lumped Mass Equivalent Systems:**

Helical Spring or Rod

\[ m_{eq} = m + \frac{1}{3} m_s \]

Cantilever Beam

\[ m_{eq} = 0.24 m_{beam} \]

Simply supported beam

\[ m_{eq} = 0.5 m_{beam} \]

Shaft in Torsion

\[ J_{eq} = J/3 \]

Others in Cochin and Plass: Bellows, clock springs, fluid masses, etc.
Lumped Stiffness Equivalent Systems:

Rod in tension/compression (A=area, E = Young’s Modulus, L = length)

\[ k_{eq} = \frac{EA}{L} \]

Rod in Torsion (D = diameter, G = shear modulus, L = length)

\[ k_{eq} = \frac{G\pi D^4}{32L} \]

Cantilevered beam (E = Young’s modulus, I = area moment of inertia, L = length, b = beam width, h = beam thickness)

\[ k_{eq} = \frac{3EI}{L^3} \quad \text{where} \quad I = \frac{bh^3}{12} \]

Simply supported beam

\[ k_{eq} = \frac{48EI}{L^3} \]

Double clamped beam

\[ k_{eq} = \frac{192EI}{L^3} \]

Helical wire coil in translation (round wire) (G = shear modulus, d = wire diameter, R = coil radius, N = number of coils)

\[ k_{eq} = \frac{Gd^4}{64R^3N} \]

Others in Cochin and Plass: rubber in compression, disk springs, other beams with different boundary conditions, coiled spring in torsion, clock springs, fluid springs, etc.
Lumped Damping Equivalent Systems

Translating Surface (sliding viscous friction)

\[ D_{eq} = \frac{\mu A}{h} \]
where
- \( A = \) wetted surface area
- \( \mu = \) coefficient of viscosity (absolute)

Rotating disk

\[ D_{eq} = \frac{\pi \mu D^4}{32h} \]

Rotating cylinder (journal bearing)

\[ D_{eq} = \frac{\pi \mu D^3 L}{4h} \]

Others in Cochin and Plass: Translating cylinder, damping due to flow restriction (dashpots), damping due to drag, mechanical dampers, structural damping, rubber pads, etc.