Lab 5: Tank-Draining Experiment

Initial-condition response, parameter identification, and model validation

Objectives

The goals of this experiment are:

- To measure the initial-condition response of a single draining tank, as shown in Fig. 1.
- To compare this experimental response to a model prediction and to use this comparison to obtain a best estimate the value of the discharge coefficient $C_d$, for each of two tanks.
- To use the best-estimate single-tank models to predict the behavior of the two-tank system shown in Fig. 2, and to compare this prediction to the experimental initial-condition response of the two-tank system.

Location

Four tank-draining experimental stations are located in the Controls Laboratory, Room C116. The lock combination is 2&4-3. Bring a floppy disk. This lab may only be used for work related to this class and other specified classes, so do not tell anyone else the lock combination. ES205 students are permitted to use the lab 24/7 except when another class is using the room.

Outline of main tasks

In this lab, you will accomplish the following tasks.

1. Calibrate the pressure transducers for each tank.
2. Experimentally obtain the initial-condition response of each tank, and by comparing the experimental response to the predicted response of your mathematical model, determine a best estimate of the orifice discharge coefficients.
3. Experimentally obtain the initial-condition response of a two-tank system and compare this response to the simulation results of a Simulink model.
MODELS

Derive the nonlinear differential equation describing the height, $h_1$, of the liquid for the upper tank assuming the liquid level is originally $h_1(0)$ before the orifice is opened. This equation should be in terms of $A_1$, $A_{o1}$, $C_{d1}$, $t$, $h_{ic}$ and the gravitational constant $g$. Solve this ODE for $h_1(t)$ (exact solution of a nonlinear ODE). Repeat for the lower tank.

Derive the nonlinear differential equations for the two-tank system and develop a Simulink model for solving the equations of motion. The model should allow you to set a different initial condition (initial water height) for each tank. Simulation results (output) is the water height as a function of time.

EXPERIMENTAL PROCEDURE

The tanks are located at stations 2, 5, 8, and 11. Record your station and identify it in your report. Bring a scale (ruler) to the lab to take measurements necessary to determine the tank areas $A_1$ and $A_2$, the orifice cross-sectional areas $A_{o1}$ and $A_{o2}$, and the height of the orifices above the tank bottom.

Some notes on operating the equipment

1. Before turning power on, make sure that the water flow-path is complete. Check the valve positions, tube positions, drain connections, and so forth.
2. Before turning the rack power on, make sure that all unused current modules have resistor-plugs.
3. If you turn on the power and a pump starts, it was left switched on after its last use. Just turn the pump switch off, and after starting one of the computer programs you can turn the pump switch back on (“forward”).
4. Each apparatus has two pumps, distinguished by a wrap of either red or white tape. The banana-plug connectors on the rack to power each pump are color-coded with matching red and white tape.
5. The data acquisition system is set up to read only one pressure transducer at a time. Connect the data cable to the correct output.
6. The pressure transducer measures the pressure at the bottom of the tank.
7. When finished with the apparatus, turn off the power to the rack, switch the pump off, remove your files from the computer hard-drive, and turn off the computer.

Calibration

Calibrate the pressure transducers using the program TANKCAL.EXE located in F:\CLASS. (A “shortcut to class” folder is on the computer desktop.) A sensor calibration requires a relationship between the sensor input (physical variable) and the sensor output (electrical signal). Hopefully this relationship is linear or easily understood so that the collected electrical data can be converted back to a physical variable.
Power up the equipment. A shortcut to the tank-calibration program (TANKCAL) is located on the desktop. Run TANKCAL. The computer needs to be connected to the pump and the pressure sensor as demonstrated in class. Fill the tank to about 11 inches of water (use your finger to plug the orifice). There is a ruler in the tank which can be used to determine the height of the water. The voltage corresponding to this height is shown on the computer screen. Gradually let out the water and collect data in a table similar to the one shown below (again, use your finger to plug the orifice to take a reading). Take as many data points as you think are necessary, creating a table similar to Table 1. The minimum height measured should be about 3 inches. From your calibration graph, obtain the slope ($m$) and y-intercept ($b$) of the linear curve relating measured voltage to water height.

<table>
<thead>
<tr>
<th>Data point no. $i$</th>
<th>Water height from bottom (in)</th>
<th>Transducer output (V)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>11.0</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>:</td>
<td></td>
</tr>
<tr>
<td>...</td>
<td>:</td>
<td></td>
</tr>
<tr>
<td>$n$</td>
<td>:</td>
<td></td>
</tr>
</tbody>
</table>

**Initial condition response**

Use the data acquisition software TANK to collect data for the transient IC response. Use the “shortcut to TANK” icon on the computer desktop. When TANK launches, follow the screen instructions. Fill the tank to about 11 inches and obtain the initial condition response. The voltage values are written to a data file named ALOTA.DAT (in the C:\user directory). These data are in comma-delimited columns. Before running the program again, find the file and change its name so it is not overwritten the next time the program is run.

**Repeat**

Repeat the initial-condition experiment for the second tank.

**Two-tank experiment**

Fill the upper tank to a fluid height of approximately 10 inches and the lower tank to a fluid height of approximately 5 inches (all heights measured from the bottom of the tanks). The upper tank drains into the lower tank. Both orifices are opened at the same time. Using TANK, record the height of fluid in the lower tank as a function of time until the lower tank water height has dropped to at least three inches. *Make sure you are recording the sensor output from the correct transducer.*

**When you are finished**

Save your data files to your own floppy disk and delete the files from the hard-drive of the data-acquisition computer. Turn off the computer, the pump, and the power to the apparatus.
ANALYSIS AND SIMULATION

Parameter identification: determine $C_d$ for each tank

For your initial analysis assume the discharge coefficient $C_d$ is approximately equal to 0.7. Substitute this value of $C_d$ and the parameters you measured into your analytical expression for $h_1(t)$ (from your exact solution of the nonlinear differential equation). Using this model, compute the predicted response for $h_1(t)$ for the same time span as the experiment. Plot this prediction and the experimental data on the same figure. Use a continuous line for the theoretical prediction and individual symbols for the experimental data.

Tuning the model

Generally, analytical solutions and experimental measurements differ. The theoretical curve can be made to more closely (though usually not exactly) predict the experimental measurements by adjusting the model parameters (in this case, the discharge coefficient $C_d$). A measure of experimental/theoretical closeness is the performance index $J$ given by

$$J = \frac{1}{n} \sum_{i=1}^{n} (h_{i}^{\text{model}} - h_{i}^{\text{data}})^2$$

where:

$h_{i}^{\text{model}}$ = water height predicted by the model at the $i^{th}$ point in time

$h_{i}^{\text{data}}$ = water height experimentally determined at the $i^{th}$ point in time

$n$ = number of data points used in comparing experiment to theory

Your task is to find the value of $C_d$ that minimizes $J$. One possible procedure is:

- Guess a value for $C_d$ and compute $J$ (this could be done in Matlab or Excel).
- Repeat for a range of $C_d$ values
- Plot $J$ as a function of $C_d$ and determine the value of $C_d$ that minimizes $J$.
- Repeat this process to determine the discharge coefficient of the lower tank.

Predicting the response of the two-tank system

Use your Simulink model of the two-tank system to predict the height in the lower tank as a function of time, and predict how long it takes for the lower tank to drain to a level of 4 inches of fluid if the upper tank is initially filled to a fluid height of 10 inches and the lower tank is initially filled to a fluid height of 5 inches (all heights measured from the bottom of the tanks). The upper tank drains into the lower tank. Both orifices are opened at the same time. Compare the simulation to the results of the two-tank experiment. Make sure your model initial conditions match the experimental initial conditions.
REPORTING

Two reports are due. The first is a memo-style progress report due one week after your first tank-draining lab day. The second is a formal lab report due two weeks later. (Check the course calendar.) Both reports are team-written and a team grade will be assigned.

The memo-style progress report shall report and provide some discussion of the following:
1. Final value of the discharge coefficients for your upper and lower tanks.
2. For each tank, a plot of the experimental data and the theoretical prediction for the initial-condition response.
3. A plot of the experimental data, lower tank, for the two-tank experiment.

The formal lab report shall include all the results described above plus all appropriate sections described in the writing standards, e.g., objectives, procedure, and so forth. The formal lab report must include the following results as a minimum. Figures are to be embedded in the body of the report and thoroughly discussed. Use different line types and legends; plot experimental data as data points and theoretical responses as continuous lines.
• Calibration curve for each tank.
• For one tank, a plot of the experimental initial-condition response, the theoretical prediction before tuning, and the theoretical prediction after tuning, all on the same graph. In another graph, repeat for the second tank.
• For the two-tank experiment, compare the predicted value to the experimental value of the time required for the lower tank to reach a fluid level of 4 inches.
• For the two-tank experiment, a plot comparing the theoretical (simulation) to experimental height $h_2(t)$ of the fluid in the lower tank as a function of time.

Notes on other sections
Appendices can include:
• Derivation of the equations of motion
• Copy of the Simulink model, m-files, and Maple worksheets.

Theory section should include:
• System schematic
• Model of the single tank and the analytical solution for this model
• Model of the two-tank system
• A table listing system parameters (constants such as area, $C_d$, and so forth) used in the models.