Example Problem - Le 13

Ex. Arm OB of the linkage has a clockwise angular velocity of 10 rad/s in the position shown where \(\theta = 45^\circ\). Determine:
(a) the velocity of point A,
(b) the velocity of point D,
(c) the angular velocity of link AB
(taken from Engineering Mechanics, 3rd Edition by Meriam & Kraige)

Vector Approach (Relative Motion)

Strategy:
1. Solve for \(\vec{V}_B\) knowing \(\vec{O}_B\) and \(\vec{r}_{B/O}\)
2. Knowing \(\vec{V}_B\) and \(\vec{r}_{A/B}\), solve for \(\vec{V}_A\) and \(\vec{O}_{AB}\)
3. Knowing \(\vec{V}_B\) and \(\vec{r}_{D/B}\), solve for \(\vec{V}_D\)

Part 1:
\[
\vec{V}_B = \vec{V}_O + \vec{O}_B \times \vec{r}_{B/O}
\]
Since O is hinged and therefore the point of rotation, \(\vec{V}_O = 0\). From the diagram, \(\vec{O} = -10\hat{k}\, \text{rad/s}\) and \(\vec{r}_{B/O} = -6\hat{i} + 6\hat{j}\, \text{in.} \). Thus
\[
\vec{V}_B = (-10\hat{k}) \times (-6\hat{i} + 6\hat{j}) = 60\hat{i} + 60\hat{j}\, \text{in/s}
\] (1)

Part 2:
\[
\vec{V}_A = \vec{V}_B + \vec{O}_{AB} \times \vec{r}_{A/B}
\]
\[
v_{A,x} = v_{B,x} - \omega_{AB} r_{A/B,y}
\]
\[
v_{A,y} = v_{B,y} + \omega_{AB} r_{A/B,x}
\]
From the diagram, \(v_{A,y} = 0\) and \(\vec{r}_{A/B} = -14\hat{i} + 0\hat{j}\, \text{in.} \). Thus we can write the last equation from above in component form:
\[
\hat{i}: \quad v_{A,x} = v_{B,x} - \omega_{AB} r_{A/B,y}
\]
\[
\hat{j}: \quad v_{A,y} = v_{B,y} + \omega_{AB} r_{A/B,x}
\]
\[
0 = v_{B,y} + \omega_{AB} (-14)
\] (3)
Solving the two equations (2,3) for the two unknowns \((v_{A,x}, \omega_{AB})\):
\[
v_{A,x} = 60\, \text{in/s}, \quad v_{A,y} = 0\, \text{in/s} \quad \Rightarrow \quad \vec{V}_A = 60\hat{i}\, \text{in/s}
\]
\[
\omega_{AB} = 4.28 \quad \Rightarrow \quad \vec{O}_{AB} = 4.28\hat{k}\, \text{rad/s}
\]

Part 3:

Vector Algebra Example
\( \bar{v}_D = \bar{v}_B + \omega_{AB} \times \bar{r}_{D/B} \\
= v_{B,x} \hat{i} + v_{B,y} \hat{j} + \left( \omega_{AB} \hat{k} \right) \times \left( r_{D/B,x} \hat{i} + r_{D/B,y} \hat{j} \right) \)

\[ v_{D,x} \hat{i} + v_{D,y} \hat{j} = v_{B,x} \hat{i} + v_{B,y} \hat{j} - \omega_{AB} r_{D/B,x} \hat{j} + \omega_{AB} r_{D/B,y} \hat{i} \]

From the diagram, \( \bar{r}_{D/B} = -8\hat{i} + 0\hat{j} \text{ in} \). Thus we can write the last equation from above in component form:

\[ \hat{i} : \quad v_{D,x} = v_{B,x} - \omega_{AB} r_{D/B,y} \]
\[ v_{D,x} = v_{B,x} - 0 \quad (4) \]

\[ \hat{j} : \quad v_{D,y} = v_{B,y} + \omega_{AB} r_{D/B,x} \]
\[ v_{D,y} = v_{B,y} + 4.28(-8) \quad (5) \]

Solving the two equations (4,5) for the two unknowns \((v_{D,x}, v_{D,y})\):

\[ v_{D,x} = 60 \text{ in/s}, \quad v_{D,y} = 25.76 \text{ in/s} \]

\[ v_D = 60\hat{i} + 25.76\hat{j} \text{ in/s} \]

which is identical to the result obtained using the instantaneous center of velocity and the scalar approach.