This exam is closed-book in nature. You are not to use a calculator or computer during the exam. Do not write on the back of any page, use the extra pages at the end of the exam. You must show your work to receive credit for a problem.
1. **(35 points)** Assume \( x(t) = 4 \text{sinc} \left[ \frac{1}{\pi} (t - 2) \right] \cos(4(t - 2)) \) is the input to an LTI system with transfer function
\[
H(\omega) = \begin{cases} 
\frac{1}{\pi} e^{-j\omega} & |\omega| > 4 \\
0 & \text{else}
\end{cases}
\]
a) Determine the Fourier transform \( X(\omega) \) of \( x(t) \)
b) Accurately sketch the magnitude and phase of \( X(\omega) \)
c) Determine the energy in \( x(t) \)
d) Sketch the magnitude and phase of \( Y(\omega) \)
e) Determine the system output \( y(t) \)
2. **(30 points)** Fill in the following table, show all your work

<table>
<thead>
<tr>
<th>$x(t)$</th>
<th>$X(\omega)$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\left[ \text{rect} \left( \frac{\omega + 2}{3} \right) + \text{rect} \left( \frac{\omega - 2}{3} \right) \right] e^{j\omega 3}$</td>
</tr>
<tr>
<td></td>
<td>$\frac{3}{2 + j\left( \frac{\omega}{5} + 2 \right)}$</td>
</tr>
<tr>
<td>$\frac{4}{4 + (2t - 4)^2}$</td>
<td></td>
</tr>
</tbody>
</table>
3. **(20 points)** Consider the signal \( x(t) = \cos(4t) + \cos(6t) \)

a) Sketch the spectrum of \( X(\omega) \)

b) \( x(t) \) is the input to an ideal sampler sampling at rate \( f_s = \frac{5}{2\pi} = \frac{1}{T} \) seconds. Sketch the spectrum of the sampled signal \( X_s(\omega) \).

c) Assume \( x_s(t) \) is the input to an ideal lowpass filter with a cutoff frequency of 7 rad/sec and passband gain of \( T \). Determine the output signal \( x_r(t) \) and write it in terms of the original signal \( x(t) \) plus any aliased terms.
4) **(15 points)** The periodic signal $x(t)$ has the Fourier series representation

$$x(t) = 2 + \sum_{k=-\infty}^{\infty} \frac{1}{1 + kj} e^{jk3t}$$

$x(t)$ is the input to an LTI system (a high pass filter) with the transfer function

$$H(j\omega) = \begin{cases} 
0 & |\omega| < 2 \\
4e^{-j2\omega} & |\omega| > 2 
\end{cases}$$

The steady state output of the system can be written as

$$y(t) = ax(t - b) + c + d \cos(e(t - b) + f).$$

Determine the output, writing it in as simple a form (like that above) as you can.
Some Potentially Useful Relationships

\[ E_\infty = \lim_{T \to \infty} \int_{-T}^{T} |x(t)|^2 dt = \int_{-\infty}^{\infty} |x(t)|^2 dt \]

\[ P_\infty = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} |x(t)|^2 dt \]

\[ e^{jx} = \cos(x) + j\sin(x) \quad j = \sqrt{-1} \]

\[ \cos(x) = \frac{1}{2} [e^{jx} + e^{-jx}] \quad \sin(x) = \frac{1}{2j} [e^{jx} - e^{-jx}] \]

\[ \cos^2(x) = \frac{1}{2} + \frac{1}{2} \cos(2x) \quad \sin^2(x) = \frac{1}{2} - \frac{1}{2} \cos(2x) \]

\[ \text{rect} \left( \frac{t - t_0}{T} \right) = u \left( t - t_0 + \frac{T}{2} \right) - u \left( t - t_0 - \frac{T}{2} \right) \]