Similarly for reflected wave

\[ V^-(z) = \Gamma_L V^+(z) = \frac{\Gamma_L V_0}{1 - \Gamma_L^2 e^{-j 2\beta_0}} e^{j\beta z} \]

**Reflection Coefficient**

\[ \Gamma(z) = \frac{V^-(z)}{V^+(z)} = \Gamma_L e^{-j 2\beta z} \]

**Phasor Additions**

Voltage varies along line

---

Figure 15 - Standing Wave Plots for Various Loads.

<table>
<thead>
<tr>
<th>( Z_L )</th>
<th>Open</th>
<th>Short</th>
<th>((0.6+j0.8)Z_C)</th>
<th>((0.92-j0.39)Z_C)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \Gamma_L )</td>
<td>+1</td>
<td>-1</td>
<td>0.5( \angle \pi/2 )</td>
<td>0.2( \angle -\pi/2 )</td>
</tr>
<tr>
<td>VSWR</td>
<td>( \infty )</td>
<td>( \infty )</td>
<td>3</td>
<td>1.5</td>
</tr>
<tr>
<td>( (d-z)_{MAX} )</td>
<td>0+( n\lambda/2 )</td>
<td>( \lambda/4+n\lambda/2 )</td>
<td>( \lambda/8+n\lambda/2 )</td>
<td>3( \lambda/8+n\lambda/2 )</td>
</tr>
<tr>
<td>( (d-z)_{MIN} )</td>
<td>( \lambda/4+n\lambda/2 )</td>
<td>0+( n\lambda/2 )</td>
<td>3( \lambda/8+n\lambda/2 )</td>
<td>( \lambda/8+n\lambda/2 )</td>
</tr>
</tbody>
</table>

Table 1 - Summary of Standing Wave Plots of Figure 53.
Figure 14 - Variation of Incident and Reflected Phasors with Distance.

\[
Z(z) = \frac{V(z)}{I(z)} = \frac{V^+(z) + V^-(z)}{I^+(z) + I^-(z)}
\]

\[
= \frac{V_o^+ (e^{-j\beta z} + \Gamma_L e^{-j\beta (d-z)})}{Z_c} \frac{V_o^+ (e^{-j\beta z} - \Gamma_L e^{-j\beta (2d-z)})}{Z_c}
\]

\[
= Z_c \left[ \frac{e^{j\beta (d-z)} + \Gamma_L e^{-j\beta (d-z)}}{e^{j\beta (d-z)} - \Gamma_L e^{-j\beta (d-z)}} \right]
\]
\[ Z_{\text{IN}}(d) = Z_c \frac{Z_L + jZ_c \tan \beta d}{Z_c + jZ_L \tan \beta d} \]

\[ Z_{\text{IN \, MATCH}} = Z_c \]

\[ Z_{\text{IN}} \left( \frac{nL}{2} \right) = Z_L \]

\[ Z_{\text{IN}} \left( \frac{(2n-1)L}{4} \right) = \frac{Z_c}{Z_L} \]

\[ Z_{\text{IN \, SC}} = jZ_c \tan \beta d \]

\[ Z_{\text{IN \, OC}} = \frac{-jZ_c}{\tan \beta d} \]

---

**Figure 16** - Input Reactance of Short-circuit and Open-circuit Lines.
\[
VSWR = \left| \frac{V_{\text{MAX}}}{V_{\text{MIN}}} \right| = \frac{1+|\Gamma_L|}{1-|\Gamma_L|}
\]

\[
VSWR_{OC} = VSWR_{SC} = \infty
\]

\[
VSWR_{\text{MATCH}} = 1 \quad \text{"FLAT" LINE}
\]

**EXPERIMENT WITH PHASOR DEMO ON WEB**

**CRANK DIAGRAM**

**VARIATION OF VOLTAGE AND CURRENT PHASORS WITH POSITION ON LINE.**

\[
V(d) = V_{\text{INC}} + \Gamma_L e^{-j2\beta d} V_{\text{INC}}
\]

\[
= V_{\text{INC}} (1 + \Gamma_L e^{-j2\beta d})
\]

---

**Diagram:**

- **V_L**
- **\(\Gamma_L\)**
- **\(V(d_{\text{MAX}})\)**
- **\(V(d_{\text{MIN}})\)**
- **\(2\beta d_0\)**
- **\(V_{\text{INC}} = 1 \leq 0^\circ\)**
- **\(V(d_0)\)**

**a: Voltage Phasors**
\[ I(d) = \frac{V_{inc}}{Z_c} - \Gamma_L e^{-\jmath \varphi d} \frac{V_{inc}}{Z_c} \]
\[ = \frac{V_{inc}}{Z_c} \left( 1 - \Gamma_L e^{-\jmath \varphi d} \right) \]

b: Current Phasors

**COMBINE THESE TWO DATA ON ONE CHART - SMITH CHART**

A **REFLECTION COEFFICIENT CHART**.

\[ \Gamma = \frac{Z-Z_c}{Z+Z_c} = \frac{\frac{Z}{Z_c} - 1}{\frac{Z}{Z_c} + 1} = \frac{3-1}{3+1} \]

\[ Z = \frac{1+\Gamma}{1-\Gamma} \]

FORMS BASIS FOR SMITH CHART
Figure 22 - Transformations between Reflection Coefficient and Impedance Planes.

Figure 23 - Reflection Coefficient Loci for Constant Resistances.

Figure 24 - Reflection Coefficient Loci for Constant Reactances.
The Smith Chart
Texas A & M Electromagnetics and Microwave Lab
Figure 25 - The Smith Chart.

\[ \Gamma'(d) = \Gamma_e e^{j2\beta d} = \Gamma_e e^{j\frac{4\pi d}{\lambda}} \]

The $\Gamma_e$ is rotated in **negative** (cw) direction by $\frac{4\pi d}{\lambda}$ radians.

This rotation is on constant VSWR circle.

The normalized input impedance is read from chart.

**S-parameters, HP Notes from Web**