

**1** The elliptic curve  $E : y^2 = x^3 + 1$ .

(A) Verify that  $P = (2, 3)$  is a point on  $E$ .

(B) Determine  $2P$

(C) Determine  $3P$

(D) Determine  $4P, 5P, 6P, 7P, \dots$

(E) Find  $Q$  so that  $2Q = P$ .

**2** The elliptic curve  $E : y^2 = x^3 + 4x$ .

(A) Verify that  $P = (2, 4)$  is a point on  $E$

(B) Determine  $2P$

(C) Determine  $3P, 4P, 5P, 6P, 7P, \dots$

(D) Find  $Q$  so that  $2Q = P$ .

**3** The elliptic curve  $E : y^2 = x^3 - 4x$ .

(A) Verify that  $P = (2, 0)$  and  $Q = (0, 0)$  are points on  $E$

(B) Determine  $2P$

(C) Determine  $2Q$

(D) Determine  $P + Q$

(E) Find  $R$  so that  $2R = P$ .

(F) Find  $S$  so that  $2S = Q$

(G) Find  $T$  so that  $2T = P + Q$

4 The elliptic curve  $E : y^2 = x^3 - 1296x + 11664$ .

(A) Verify that  $P = (108, 0)$  is a point on  $E$ .

(B) Determine  $2P$

(C) Determine  $3P$

(D) Determine  $4P, 5P, 6P, 7P$

(E) Find  $Q$  so that  $2Q = P$ .