MA490-01 - Number Theory II - Congruences

1 Solve for x in the following congruences:
(A) $x + 5 \equiv 3 (\text{mod } 14)$
(B) $x + 18 \equiv 7 (\text{mod } 2000)$
(C) $x + 100 \equiv 53 (\text{mod } 2, 147, 483, 647)$

2 Solve for x, or show that there are no solutions:
(A) $2x \equiv 1 (\text{mod } 93)$
(B) $3x \equiv 2 (\text{mod } 2000)$
(C) $5x \equiv 112 (\text{mod } 1776)$
(D) $7x \equiv 100 (\text{mod } 4, 004)$
(E) $101x \equiv 490 (\text{mod } 2, 147, 483, 647)$
(F) $65, 536x \equiv 16 (\text{mod } 2, 147, 483, 647)$

3 Solve for x, or show that there are no solutions:
(A) $x^2 \equiv 9 (\text{mod } 10)$
(B) $x^2 \equiv 1 (\text{mod } 30)$
(C) $x^2 \equiv 7 (\text{mod } 31)$
(D) $x^3 \equiv 1 (\text{mod } 31)$
(E) $x^3 \equiv 5 (\text{mod } 31)$
(F) $x^3 \equiv 23 (\text{mod } 31)$.
(G) $x^5 \equiv 1 (\text{mod } 2, 147, 483, 647)$
(H) $x^5 \equiv 2 (\text{mod } 2, 147, 483, 647)$
(I) $x^7 \equiv 1 (\text{mod } 2, 147, 483, 647)$
(J) $x^7 \equiv 2 (\text{mod } 2, 147, 483, 647)$
(K) $x^7 \equiv 3 (\text{mod } 2, 147, 483, 647)$

4 Find the smallest positive integer n satisfying the congruence, or show that there are no solutions:
(A) $2^n \equiv 1 (\text{mod } 7)$
(B) $2^n \equiv 3 (\text{mod } 31)$
(C) $2^n \equiv 1 (\text{mod } 101)$
(D) $2^n \equiv 7 (\text{mod } 2001)$
(E) $2^n \equiv 1 (\text{mod } 2, 147, 483, 647)$
(F) $3^n \equiv 1 (\text{mod } 2, 147, 483, 647)$