Binomial Examples

Example A person trying to hit a target records 45 hits in 100 tries. Determine a 95% confidence interval for the expected number of successful hits.

Solution The estimated probability of success is $p = 45/100 = .45$. The estimated variance is $V(X) = np(1-p) = 100(.45)(.55) = 24.75$.

The standard deviation is $\sqrt{24.75} \approx 4.975$.

Our 95% confidence interval will be $(45 - 2(4.975), 45 + 2(4.975)) = (35.050, 54.950)$.

Example Create a 95% confidence interval for the person's probability of success.

Solution The estimated probability of success is $p = 45/100 = .45$. The estimated variance is $V(X) = p(1-p)/n = (.45)(.55)/n = .002475$.

The standard deviation is $\sqrt{.002475} \approx .04975$. Our 95% confidence interval will be $(.45 - 2(.04975), .45 + 2(.04975)) = (.35050, .54950)$.

Example A second person makes 100 attempts to hit the target and is successful 33 times. Determine a 95% confidence interval for the expected difference in the number of successes achieved by the first and second person.

Solution We have $X_1 = 45$, $s_1 \approx 4.975$.

For the second person, $X_2 = 33$, and $s_2 = \sqrt{100(.33)(.67)} = \sqrt{22.11} \approx 4.702$.

Our estimate of the difference is $X_1 - X_2 = 45 - 33 = 12$.

We add the variances to get the combined variance $V = 24.75 + 22.11 = 46.86$. Thus the standard deviation is $s \approx 6.845$.

The 95% confidence interval is $(12 - 2(6.845), 12 + 2(6.845)) = (-1.691, 25.691)$.

Note that the fact that zero is in the interval means that there is not a statistically significant difference between the two scores.

Example A third person makes 200 attempts to hit the target and is successful 70 times. Determine a 95

Solution For the first person, the estimated probability of success is $p_1 = .45$, with variance $V_1 = .002475$.

For the third person the estimated probability of success is $p_2 = 70/200 = .35$, with variance $V_2 = p(1-p)/n = (.35)(65)/200 = .0011375$.

The estimated difference is $p_1 - p_2 = .45 - .35 = .10$. The combined variance is $V = .002475 + .0011375 = .0036125$, giving a standard deviation of $s \approx .0601$.

The 95% confidence interval is $(.10 - 2(.0601), .10 + 2(.061)) = (-.0202, .2202)$.

Again, the difference between success rates is not statistically significant.

We may also look at the number of standard deviations from 0; $p/s = .10/.0601 \approx 1.66$. Since this indicates that the estimate is less than two standard deviations from zero, the difference between success rates is not statistically significant.