1 (A) The probability that the team that wins game 1 also wins games 2, 3, and 4 is \((1/2)^3 = 1/8\).

(B) The series is an arrangement of 4 wins, W, and 3 losses, L, with the last two games as WL. For the other 5 games, there are \(\binom{5}{3} = 10\) ways to arrange the three wins. This is out of a possible \(2^5 = 32\) arrangements. Therefore the probability is \(10/32 = 5/16\).

Note that the probability that the series lasts 4, 5, 6, 7 games is \(\binom{3}{3} \cdot \binom{4}{3} \cdot \binom{5}{3} \cdot \binom{6}{3} \), resp.

2 \((\Rightarrow)\) if \(k|(A - B)\) then \(A - B = kN\) for some \(N \in \mathbb{Z}\).

By the division algorithm, \(A = kq_A + r_A\), and \(B = kq_B + r_B\) with \(0 \leq r_A, r_B < k\). Then \(A - B = k(q_A - q_B) + (r_A - r_B)\), where \(-k < r_A - r_B < k\). Therefore \(r_A - r_B = 0\), and \(r_A = r_B\). Thus \(k|a \Leftrightarrow k|B\).

\((\Leftarrow)\) If \(k|A \Leftrightarrow k|B\), then \(A = kq_A + r_A\), and \(B = kq_B + r_B\), with \(0 \leq r_A, r_B < k\). Then \(A - B = k(q_A - q_B) = (r_A - r_B)\), so \((k|A \Leftrightarrow k|B) \Rightarrow k|(A - B)\).

3

\[
\begin{align*}
(A \cup B) \cap (A \cap (A \cap B)) & \quad \text{Premise} \\
(B \cup A) \cap (A \cap (A \cap B)) & \quad \text{Commutative Law} \\
(B \cup A) \cap ((A \cap A) \cap B) & \quad \text{Associative Law} \\
([B \cup A] \cap A) \cap B & \quad \text{Idempotent} \\
[A \cap (B \cup A)] \cap B & \quad \text{Associative} \\
[(A \cap B) \cup (A \cap A)] \cap B & \quad \text{Distributive} \\
[(A \cap B) \cup \emptyset] \cap B & \quad \text{Inverse} \\
[(A \cap B)] \cap B & \quad \text{Identity} \\
A \cap (B \cap B) & \quad \text{Associative} \\
A \cap B & \quad \text{Idempotent}
\end{align*}
\]

4 Base Case: \(n = 1\). We have \(\binom{2-1}{1} = \binom{2}{1} = 2 < 4 = 4^1\).

IH: \((n = k)\) We have \(\binom{2k}{k} < 4^k\).

Next case: \(n = k + 1\). We see that

\[
\binom{2(k + 1)}{k + 1} = \frac{(2k + 2)(2k + 1)}{(k + 1)(k + 1)} \binom{2k}{k} = 2 \frac{2k + 1}{k + 1} \binom{2k}{k} < 2 \frac{2k + 1}{k + 1} \cdot 4^k \quad \text{By IH}
\]

\[
< 2 \frac{2k + 2}{k + 1} \cdot 4^k = 2 \cdot 2 \cdot 4^k = 4^{k+1} = 4^n.
\]
We prove this by mathematical induction on $n$.

**Base case**: $(n = 1)$ \(3^1 - F_1 - F_3 = 3 - 1 - 2 = 0 = 5 \cdot 0\), so \(5|(3^1 - F_1 - F_3)\).

**IH**: \(5|3^k - F_k - F_{k+2}\)

**Next case**: \(n = k + 1\).

\[
3^{k+1} - F_{k+1} - F_{k+3} = 3 \cdot 3^k - F_k - F_{k-1} - F_{k+1} - F_{k+2}
\]

\[
= 3(3^k - F_k - F_{k+2}) + 2F_k - F_{k-1} + 2F_{k+2} - F_{k+1}
\]

\[
= 3(3A) + 2F_k - F_{k-1} + 2F_{k+2} - F_{k+1} \quad \text{by IH}
\]

\[
= 5(3A) + 2F_k - (F_{k+1} - F_k) + 2F_{k+2} - F_{k+1}
\]

\[
= 5(3A) + 3F_k + 2F_{k+2} - 2F_{k+1}
\]

\[
= 5(3A) + 3F_k + 2(F_{k+2} - F_{k+1})
\]

\[
= 5(3A) + 3F_k + 2F_k
\]

\[
= 5(3A) + 5F_k
\]

\[
= 5(3A + F_k)
\]

Thus, \(5|3^n - F_n - F_{n+2}\).

**Alternate proof**:

**Base cases**: \(n = 1\); \(3^1 - F_1 - F_3 = 3 - 1 - 2 = 0 = 5 \cdot 0\), so \(5|(3^1 - F_1 - F_3)\).

\(n = 2\); \(3^2 - F_2 - F_4 = 9 - 1 - 3 = 5 = 5 \cdot 1\), so \(5|(3^2 - F_2 - F_4)\).

**IH**: \(5|3^{k-1} - F_{k-1} - F_{k+1}\) and \(5|3^k - F_k - F_{k+2}\)

**Next case**: \(n = k + 1\).

\[
3^{k+1} - F_{k+1} - F_{k+3} = 9 \cdot 3^{k-1} - F_k - F_{k-1} - F_{k+2} - F_{k+1}
\]

\[
= 5 \cdot 3^{k-1} + (3^k - F_k - F_{k+2}) + (3^{k-1} - F_{k-1} - F_{k+1})
\]

\[
= 5 \cdot 3^{k-1} + 5A + 5B \quad \text{by IH}
\]

\[
= 5(3^{k-1} + A + B)
\]

Thus, \(5|3^n - F_n - F_{n+2}\).