Exam #1 - MA275 - Sample solutions

1 (A) the letters are *CCEEFFIIINNY*. There are 12 letters, 2 Cs, 2 Es, 2 Fs, 3 Is, and 2 Ns. Therefore there are $\frac{12!}{(2!)^4(3!)1!}$ ways to arrange the letters.

(B) There are 29 Rs, and 56 moves, so there are $\binom{56}{29}$ ways to choose the 29 places for the Rs.

(C) Subtract the number of paths through $(11, 14)$ from the number of paths found in (B). The number of paths through $(11, 14)$ is the product of the number of paths to $(11, 14)$, $\binom{25}{11}$, multiplied by the number of paths from $(11, 14)$, $\binom{56-25}{29-11} = \binom{31}{18}$.

Therefore the number of paths is $\binom{56}{29} - \binom{25}{11}\binom{31}{18}$.

2

$\neg p \wedge \neg q \wedge (p \vee q \vee \neg r)$	Premise
$\neg p \wedge \neg q \wedge ((p \vee q) \vee \neg r)$	Associativity of $\vee$
$(\neg p \wedge \neg q) \wedge ((p \vee q) \vee \neg r)$	Associativity of $\vee$
$[(\neg p \wedge \neg q) \wedge (p \vee q) \vee \neg r)] \vee [(\neg p \wedge \neg q) \wedge \neg r]$	Distributive Law
$[\neg(p \vee q) \wedge (p \vee q)] \vee [(\neg p \wedge \neg q) \wedge \neg r]$	DeMorgan's Law
$[F_0] \vee [(\neg p \wedge \neg q) \wedge \neg r]$	Inverse
$[(\neg p \wedge \neg q) \wedge \neg r]$	Identity
$\neg p \wedge \neg q \wedge \neg r$	Associativity of $\wedge$

3

$(\bar{A} \cup B) \cap (A \cap (A \cap B))$	Premise
$(B \cup \bar{A}) \cap (A \cap (A \cap B))$	Commutative Law
$(B \cup \bar{A}) \cap ((A \cap A) \cap B))$	Associative Law
$(B \cup \bar{A}) \cap (A \cap B)$	Idempotent
$[(B \cup \bar{A}) \cap A] \cap B$	Associative
$[A \cap (B \cup \bar{A})] \cap B$	Commutative
$[(A \cap B) \cup (A \cap \bar{A})] \cap B$	Distributive
$[(A \cap B) \cup \emptyset] \cap B$	Inverse
$[(A \cap B)] \cap B$	Identity
$A \cap (B \cap B)$	Associative
$A \cap B$	Idempotent

4 the total number of solutions is $\binom{33}{3}$. We must remove solutions with $x_2 \geq 26$. These are non-negative solutions to $x_1 + (x_2 - 26) + x_3 + x_4 = 4$. There are $\binom{7}{3}$ of these. Therefore there are a total of $\binom{33}{3} - \binom{7}{3}$ solutions.

5 There are $\binom{56}{29}$ paths in all. We apply the map described in Section 1.5 of the book to see that the paths t $(29, 27)$ that go above the line $y = x$ are in a one-to-one correspondence with the paths from $(0, 0)$ to $(26, 30)$. There are $\binom{56}{26}$ of these. therefore there are $\binom{56}{29} - \binom{56}{26}$ paths.

Note that this simplifies to $\frac{56!}{29!27!} - \frac{56!}{30!26!} = \frac{56!}{30!27!}(30 - 27) = \frac{3}{30}\binom{56}{29} = \frac{1}{10}\binom{56}{29}$.