Logic Worksheet #2 - MA275 - Sample solutions

1 Prove $[(p \to s) \wedge (q \to s) \wedge (r \to s)] \to [(p \vee q \vee r) \to s]$. Cite specific rules at each step.

(1)	$(p \to s)$	Premise
(2)	$(q \to s)$	Premise
(3)	$(p \vee q) \to s)$	(1), (2), proof by cases
(4)	$(r \to s)$	Premise
(5)	$((p \vee q) \vee r) \to s)$	(3), (4), proof by cases
(6)	$(p \vee q \vee r) \to s$	(5), Associativity of $\vee$

2 Negate: $[(p \vee q) \wedge \neg p] \to q$

$\neg\{[(p \vee q) \wedge \neg p] \to q\}$	
$\neg\{\neg[(p \vee q) \wedge \neg p] \vee q\}$	Substitution
$\neg\neg[(p \vee q) \wedge \neg p] \wedge \neg q$	DeMorgan
$[(p \vee q) \wedge \neg p] \wedge \neg q$	Double Negation
$[\neg p \wedge (p \vee q)] \wedge \neg q$	Commutativity of $\wedge$
$[(\neg p \wedge p) \vee (\neg p \wedge q)] \wedge \neg q$	Distributive law of $\wedge$ over $\vee$
$[F_0 \vee (\neg p \wedge q)] \wedge \neg q$	Inverse
$(\neg p \wedge q) \wedge \neg q$	Identity
$\neg p \wedge (q \wedge \neg q)$	Associativity of $\wedge$
$\neg p \wedge F_0$	Inverse
F_0	Domination

3 (a) Negate: $\forall x \exists y$ xy is one more than a multiple of eleven.

$\exists x \forall y$ xy is not one more than a multiple of eleven.

(b) Which statement is true for positive integers? Why?

The negation. If $x = 11$ then xy is a multiple of eleven, not one more than a multiple of eleven.

(c) Which statement is true for positive real numbers? Why?

The original statement. For any x, let $y = 12/x$. Then $xy = 1 + 1 \cdot 11$ is one more than a multiple of eleven.