

Delta function worksheet - MA222 - Rickert

**1** A spring-mass system follows the differential equation

$$y'' + y = f(t), \quad y(0) = 1, \quad y'(0) = 2,$$

where  $f(t)$  is the function represented by hitting the mass twice with a hammer, once with an impulse of 3 and the second time with an impulse of 2. The second hammer strike must happen at least one second after the first, and both must happen in no more than 10 seconds. Thus

$$f(t) = 3\delta(t - T) + 2\delta(t - U), \quad 0 \leq T < T + 1 \leq U \leq 10.$$

After the second blow from the hammer  $y(t)$  settles into a new equilibrium with period  $2\pi$  and amplitude  $A$ . Determine times  $T, U$  satisfying  $0 \leq T < T + 1 \leq U \leq 10$  so that the amplitude  $A$  is as large as possible.

Then determine  $T, U$ , so that  $A$  is as small as possible.

**2** A spring-mass system follows the differential equation

$$y'' + y = f(t), \quad y(0) = 1, \quad y'(0) = 2,$$

where  $f(t)$  is the function represented by hitting the mass twice with a hammer, once at time  $T$  with an impulse of 3 and the second time at time  $U$  with an impulse of

$$H(U - T) ((U - T)^2(15 - U)/100).$$

Both hammer strikes must happen in no more than 10 seconds. Thus

$$f(t) = 3\delta(t - T) + H(U - T) ((U - T)^2(15 - U)/100) \delta(t - U), \quad 0 \leq T \leq U \leq 10.$$

After the second blow from the hammer  $y(t)$  settles into a new equilibrium with period  $2\pi$  and amplitude  $B$ . Determine times  $T, U$  satisfying  $0 \leq T \leq U \leq 10$  so that the amplitude  $B$  is as large as possible.

Then determine  $T, U$ , so that  $B$  is as small as possible.