Quiz #2 - MA111 - September 18, 2008 - Sample solutions

1 $\lim_{x\to\infty} \dfrac{\sqrt{x^2+4}}{x}$. Plugging in gives $\frac{\infty}{\infty}$, an indeterminate form. So divide by $\frac{x}{x}$ to get $\lim_{x\to\infty} \dfrac{\sqrt{1+4x^{-2}}}{1}$. Now plugging in gives $\frac{\sqrt{1+0}}{1} = 1$.

2 $\lim_{x\to 3} \dfrac{\sin(x)}{x}$. Plugging in gives $\dfrac{\sin(3)}{3}$.
For those wishing a numerical approximation, this is about
.04704000268662240736691493426937009328231108808 4090.

3 $\lim_{x\to 3} \dfrac{x^2+x-12}{x-3}$ Plugging in gives $\frac{0}{0}$, an in determinant form. We note that $x^2 + x - 12 = (x-3)(x+4)$, so we divide by $\frac{x-3}{x-3}$ (also known as cancelling the common factor) to get $\lim_{x\to 3} \dfrac{x+4}{1}$. Plugging in gives $\frac{3+4}{1} = 7$.

4 $\lim_{x\to 4} \dfrac{1}{x-4}$. Plugging in gives $\frac{1}{0}$, so we look at the LH and RH limits.
Left-Hand Limit: $\lim_{x\to 4^-} \dfrac{1}{x-4} \to \dfrac{1}{0^-} \to -\infty$.
Right-Hand Limit: $\lim_{x\to 4^+} \dfrac{1}{x-4} \to \dfrac{1}{0^+} \to +\infty$.
Since the Left- and right-hand limits are not equal, the limit **does not exist**.

5 $\lim_{h\to 0} \dfrac{\sqrt{5x+5h}-\sqrt{5x}}{h}$. Plugging in gives $\frac{0}{0}$, an indeterminate form. Multiply by $\frac{\sqrt{5x+5h}+\sqrt{5x}}{\sqrt{5x+5h}+\sqrt{5x}}$ to get $\lim_{h\to 0} \dfrac{(5x+5h)-(5x)}{h(\sqrt{5x+5h}+\sqrt{5x})}$. This simplifies to $\lim_{h\to 0} \dfrac{5h}{h(\sqrt{5x+5h}+\sqrt{5x})}$. Divide by $\frac{h}{h}$ to get $\lim_{h\to 0} \dfrac{5}{(\sqrt{5x+5h}+\sqrt{5x})}$. Plugging in gives $\frac{5}{\sqrt{5x}+\sqrt{5x}} = \frac{5}{2\sqrt{5x}}$. (or $\frac{\sqrt{5}}{2\sqrt{x}}$.)