Systems, Accounting and Modeling Approach

Examples showing how students are taught to solve problems.

Examples*

Heat Engine Performance

Monkeys on a Rope

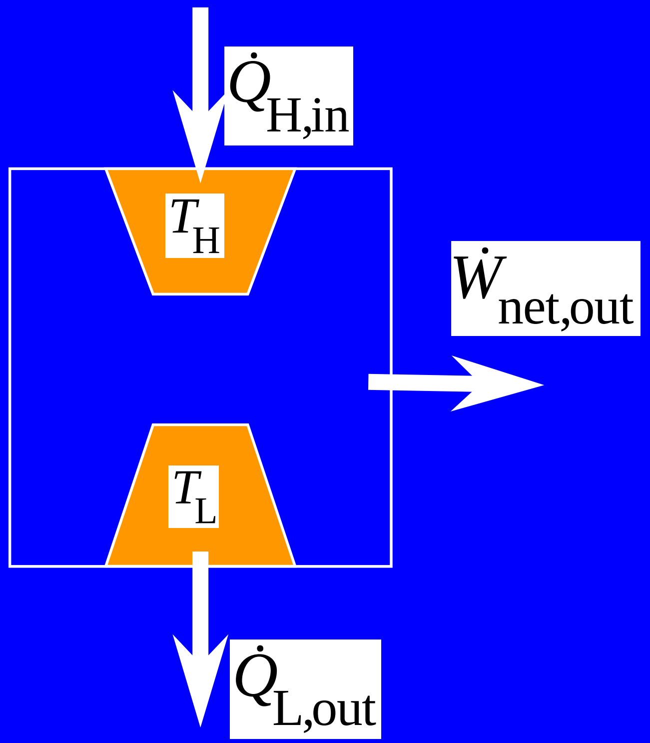
Colliding Train Cars

Grain Conveyor Belt

*Before reading the complete solution to each problem, you are encouraged to sketch out your own solution to the problem.

Example - Carnot Cycle Revisited

What is the maximum thermal efficiency of a steady-state heat engine that receives energy by heat transfer at a surface temperature T_H and rejects energy by heat transfer at a surface temperature T_L?



Analysis

What's the system?

What properties should we count?

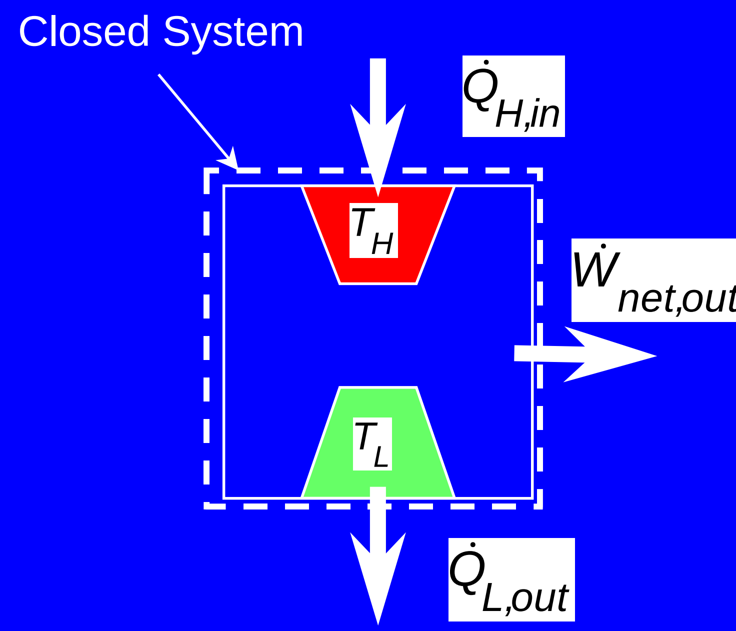
What is the time interval?

What are the important interactions?

$$\cancelto{0,SS}{\frac{dE_{sys}}{dt}} = \dot{Q}_{net,in} + \dot{W}_{net,in}$$

$$0 = \left[\dot{Q}_{H,in} - \dot{Q}_{L,out}\right] - \dot{W}_{net,out}$$

$$\boxed{\dot{W}_{net,out} = \dot{Q}_{H,in} - \dot{Q}_{L,out}}$$



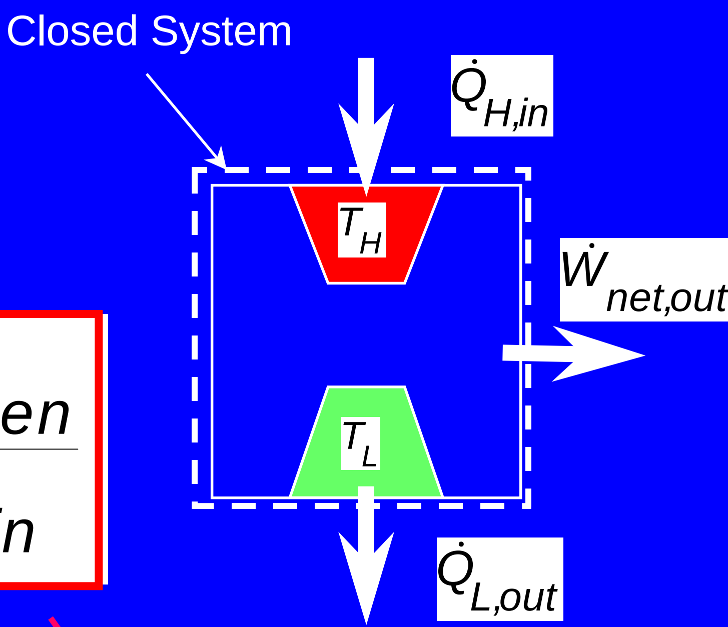
$$\cancelto{0,SS}{\frac{dS_{sys}}{dt}} = \sum_j \frac{\dot{Q}_j}{T_j} + \dot{S}_{gen}$$

$$0 = \left[\frac{\dot{Q}_{H,in}}{T_H} - \frac{\dot{Q}_{L,out}}{T_L}\right] + \dot{S}_{gen}$$

$$\boxed{\dot{Q}_{L,out} = \dot{Q}_{H,in}\left(\frac{T_L}{T_H}\right) + T_L\dot{S}_{gen}}$$

$$\dot{W}_{net,out} = \dot{Q}_{H,in} - \left[\dot{Q}_{H,in}\frac{T_L}{T_H} + T_L\dot{S}_{gen}\right]$$

$$\boxed{\eta = \frac{\dot{W}_{net,out}}{\dot{Q}_{H,in}} = \left[1 - \frac{T_L}{T_H}\right] - \frac{T_L\dot{S}_{gen}}{\dot{Q}_{H,in}}}$$



$$\eta = \frac{\dot W_{net,out}}{\dot Q_{H,in}} = \left[1 - \frac{T_L}{T_H}\right] - \boxed{\frac{T_L \dot S_{gen}}{\dot Q_{H,in}}}$$

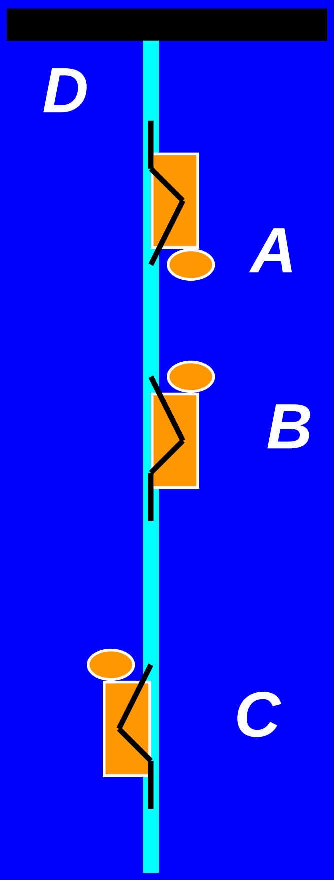
$$\eta\Big|_{max} = \left[1 - \frac{T_L}{T_H}\right]$$

$$\frac{T_L \dot S_{gen}}{\dot Q_{H,in}} \geq 0$$

Example - Monkeys on a Rope

Three monkeys *A, B, and C* with masses of 10, 12, and 8 kg, respectively, are climbing up and down the rope suspended from point *D*. At the instant shown in the figure, *A* is descending the rope with an acceleration of 1.6 m/s^2, and *C* is pulling himself up with an acceleration of 0.9 m/s^2. Monkey *B* is climbing up with a constant speed of 0.6 m/s.

Determine the tension *T* in the rope at *D,* in newtons.



Analysis

What's the system?

What properties should we count?

What is the time interval?

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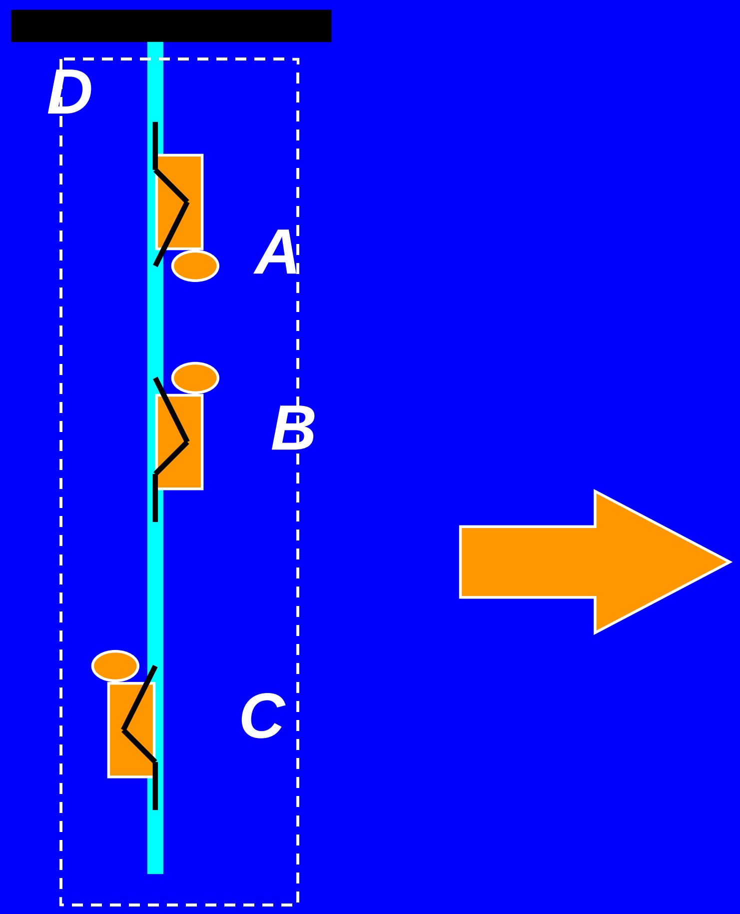
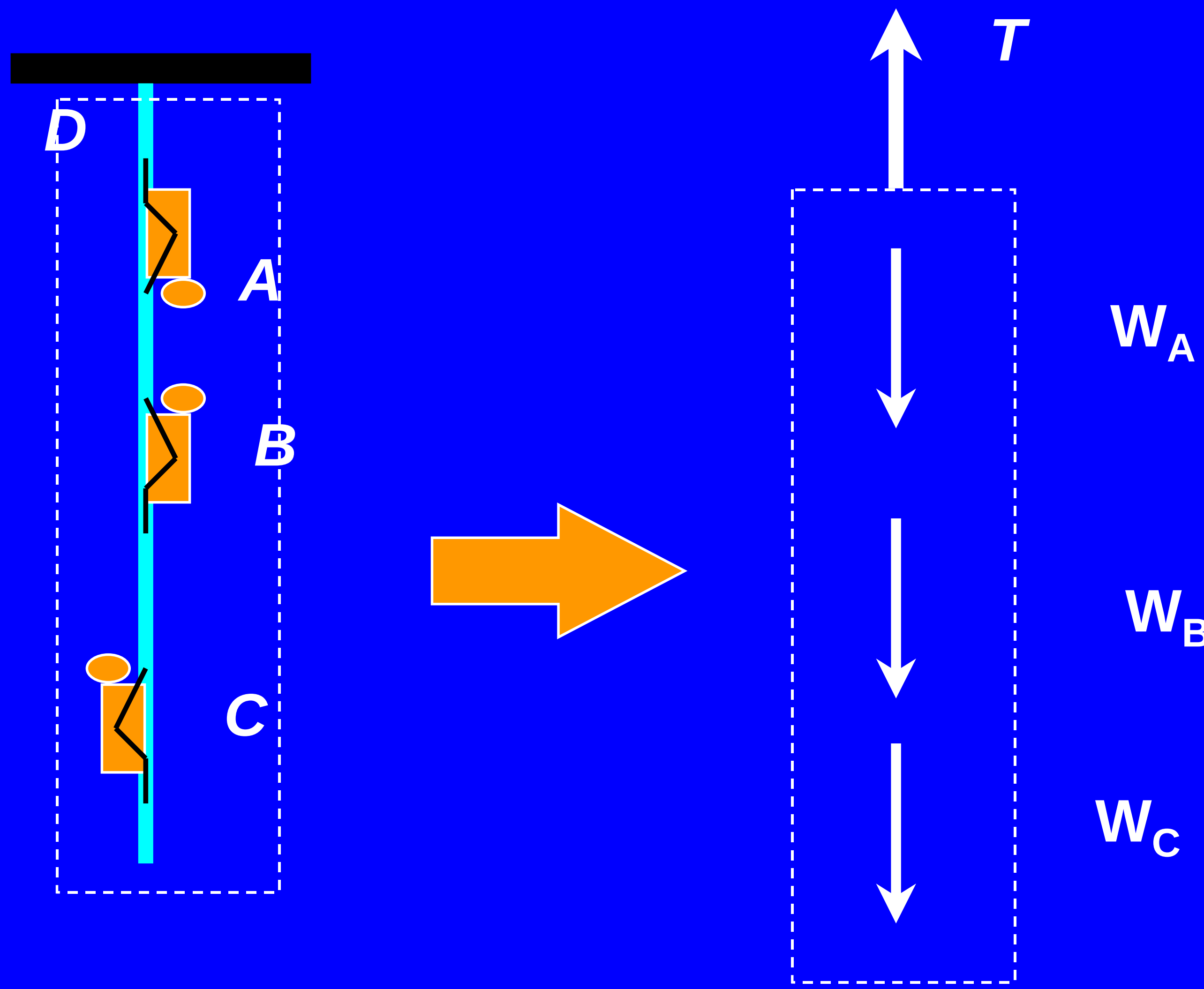


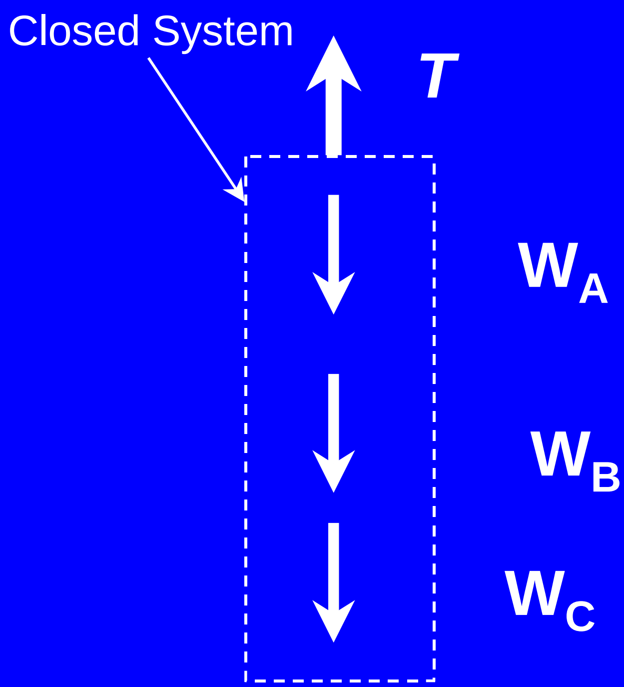
Physical System



Physical System

Free-body Diagram

$$\frac{d\vec{P}_{sys}}{dt} = \sum_j \vec{F}_{ext,j}$$



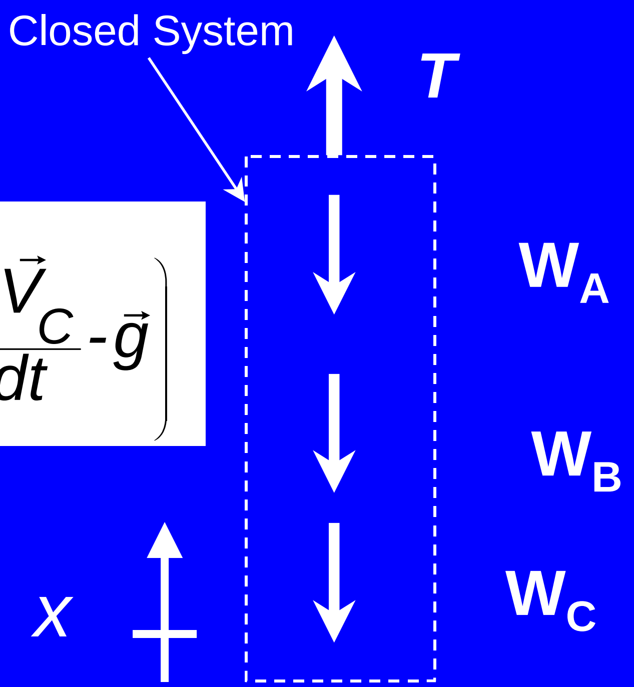
$$\sum_j \vec{F}_{ext,j} = \vec{W}_A + \vec{W}_B + \vec{W}_C + \vec{T}$$

$$\vec{P}_{sys} = m_A\vec{V}_A + m_B\vec{V}_B + m_C\vec{V}_C$$

$$\frac{d}{dt}\left(m_A\vec{V}_A + m_B\vec{V}_B + m_C\vec{V}_C\right) = \vec{W}_A + \vec{W}_B + \vec{W}_C + \vec{T}$$

$$m_A\left(\frac{d\vec{V}_A}{dt}\right) + m_B\left(\frac{d\vec{V}_B}{dt}\right) + m_C\left(\frac{d\vec{V}_C}{dt}\right) = m_A\vec{g} + m_B\vec{g} + m_C\vec{g} + \vec{T}$$

$$\vec{T} = m_A\left(\frac{d\vec{V}_A}{dt} - \vec{g}\right) + m_B\left(\frac{d\vec{V}_B}{dt} - \vec{g}\right) + m_C\left(\frac{d\vec{V}_C}{dt} - \vec{g}\right)$$



$$\begin{aligned} T &= (10\ \mathrm{kg})[-1.6+9.81]\left(\frac{\mathrm{m}}{\mathrm{s}^2}\right) + (12\ \mathrm{kg})[0+9.81]\left(\frac{\mathrm{m}}{\mathrm{s}^2}\right) + (8\ \mathrm{kg})[0.9+9.81]\left(\frac{\mathrm{m}}{\mathrm{s}^2}\right) \\ &= 82.1\mathrm{N} + 117.7\ \mathrm{N} + 85.6\ \mathrm{N} \end{aligned}$$

$$T = 285.4\ \mathrm{N}$$

Examples - Rail Cars on the Move

A 45-Mg railroad car moving with a velocity of 3 km/h is to be coupled to a 25-Mg car which is at rest.

Determine

(a) the final velocity of the coupled cars

(b) the average impulsive force acting on each car if the coupling is completed in 0.3 s.

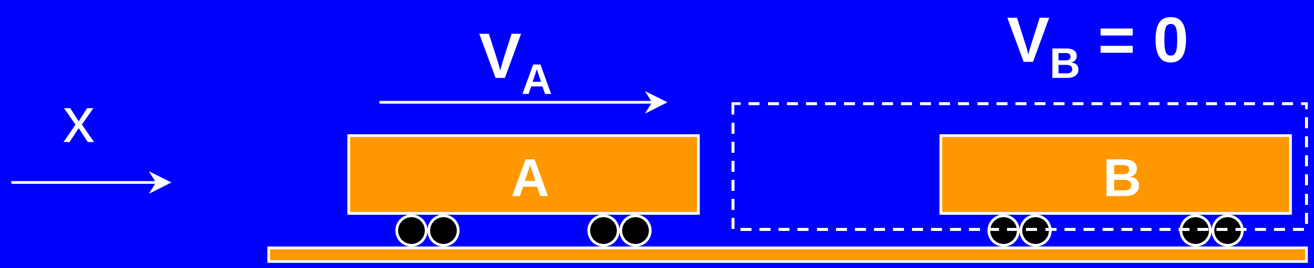
Part (a) - Final velocity after coupling

System: Assume an **open**, **moving system**.

Initially it contains car *B* only and finally it contains both cars.

Property:

$$\frac{d\vec{P}_{sys}}{dt} = \sum_j \vec{F}_{ext,j} + \sum_{in} \dot{m}_i \vec{V}_i - \sum_{out} \dot{m}_e \vec{V}_e$$



Conservation of Linear Momentum (x-direction)

$$\boxed{\frac{dP_{sys,x}}{dt} = \dot{m}_i V_{x,i}}$$

Integrate over time interval t_1 to $t_{2.}$

$$\int_{t_1}^{t_2} \left(\frac{dP_{sys,x}}{dt} \right) dt = \int_{t_1}^{t_2} \left(\dot{m}_i V_{x,i} \right) dt$$

1 - Initial state

2 - Final state

$$P_{sys,x,2} - P_{sys,x,1} = m_A V_{A,1}$$

$V_{B,1} = 0$

$$\left(m_A + m_B \right) V_{AB,2} - \cancelto{0}{m_B V_{B,1}} = m_A V_{A,1}$$

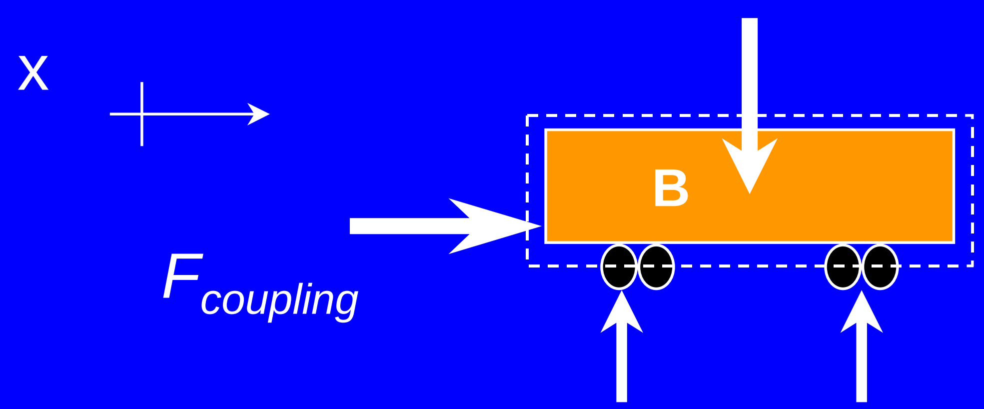
$$\boxed{V_{AB,2} = \frac{m_A V_{A,1}}{m_A + m_B}}$$

Part (b) Coupling Force

<u>System:</u> Assume a **closed system** that only contains car *B* throughout the process. This system moves with car *B*.

<u>Property:</u>

$$\frac{d\vec{P}_{sys}}{dt} = \sum_j \vec{F}_{ext,j} + \cancelto{0}{\sum_{in} \dot{m}_i \vec{V}_i - \sum_{out} \dot{m}_e \vec{V}_e}$$



Conservation of Linear Momentum (x-direction)

$$\boxed{\frac{dP_{sys,x}}{dt} = F_x}$$

Integrate over time interval t_1 to $t_{2.}$

$$\int_{t_1}^{t_2} \left(\frac{dP_{sys,x}}{dt} \right) dt = \int_{t_1}^{t_2} F_x \, dt$$

1 - Initial state

2 - Final state

$$P_{sys,x,2} - P_{sys,x,1} = F_{x,avg} \Delta t$$

$V_{B,1} = 0$

$$m_B V_{B,2} - \cancelto{0}{m_B V_{B,1}} = F_{x,avg} \Delta t$$

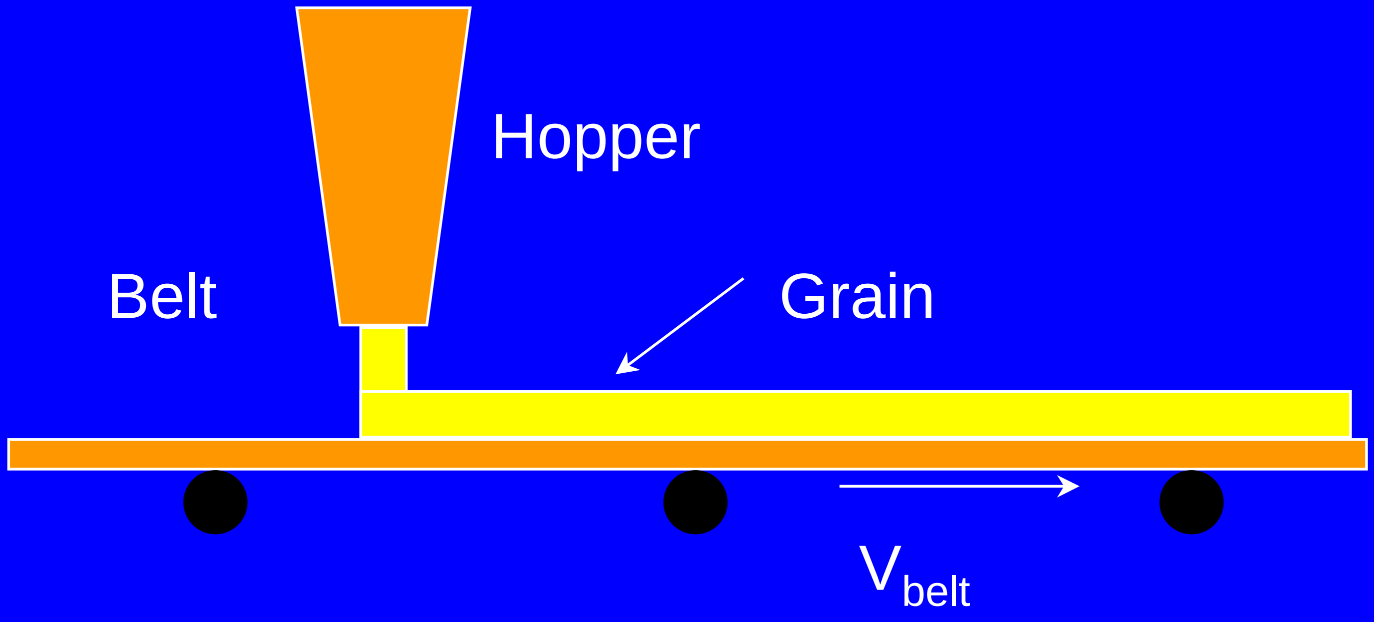
$V_{B,2} = V_{AB,2}$ from part (a)

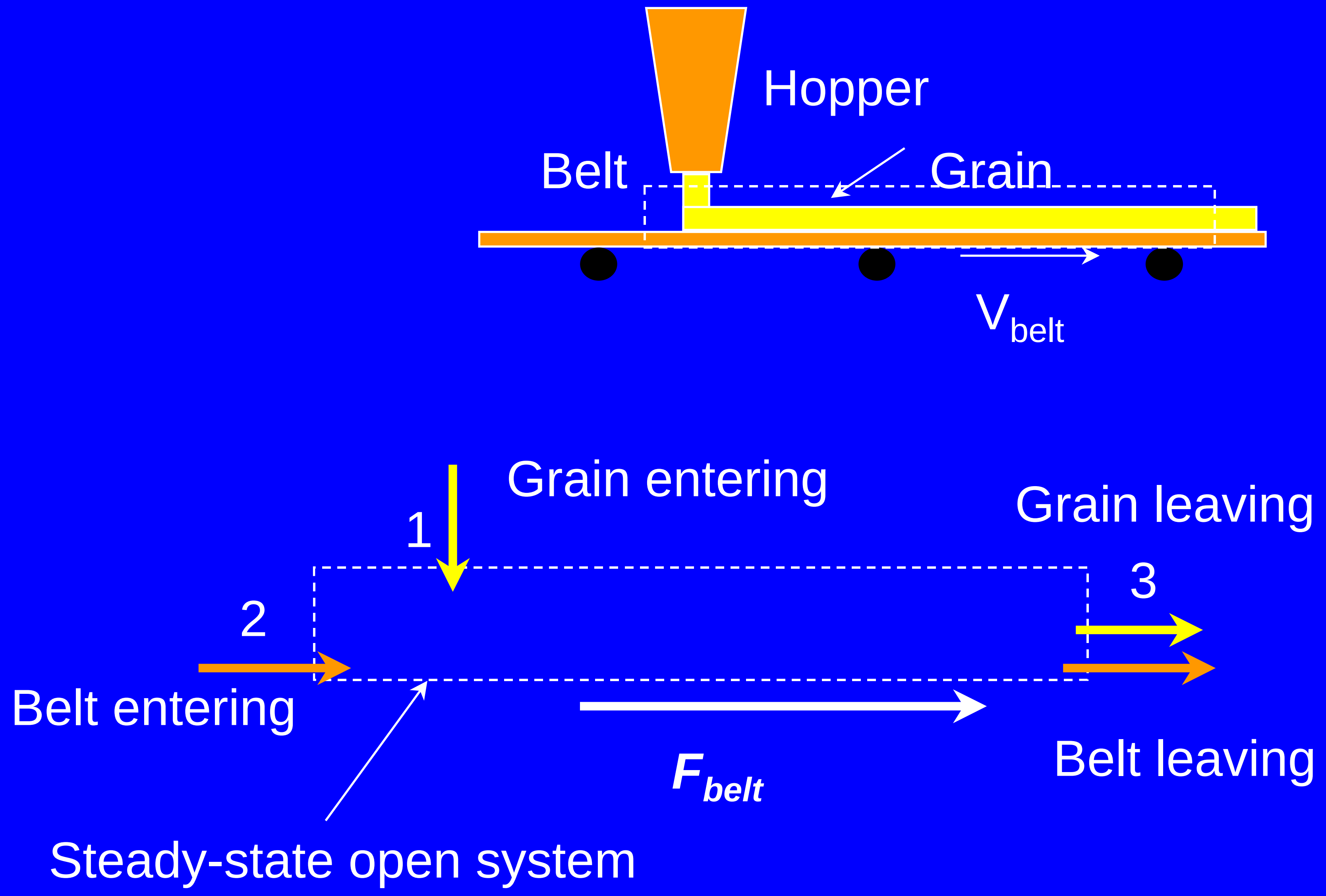
$$\boxed{F_{x,avg} = \frac{m_B V_{B,2}}{\Delta t}}$$

Example - Conveyor Belt

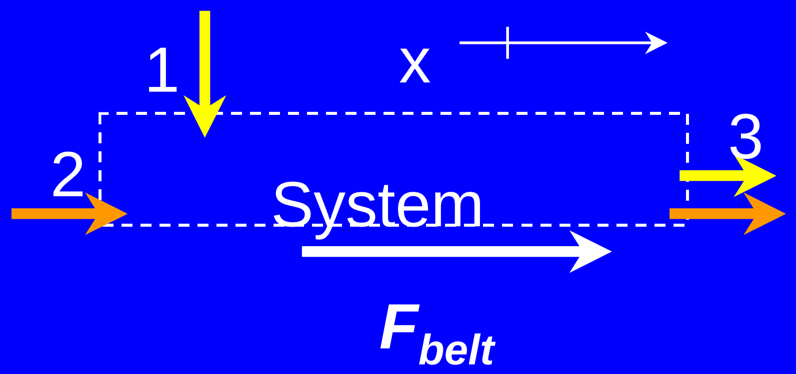
Grain falls from a hopper onto a conveyor belt at the rate of 200 kg/min. The conveyor belt carries the grain away at a constant velocity of 2 m/s.

Determine the force on the belt required to keep the belt moving at a constant speed.




Hopper
Belt
Grain
V_{belt}
Grain entering
1
Grain leaving
3
2
Belt entering
F_{belt}
Belt leaving
Steady-state open system

Conservation of mass
for this system.



$$\frac{dm_{sys}}{dt} = \dot{m}_{grain,1} + \dot{m}_{belt,2} - \dot{m}_{belt,3} - \dot{m}_{grain,3}$$

$$m_{sys} = m_{belt,sys} + m_{grain,sys}$$

0, SS

$$\frac{dm_{belt,sys}}{dt} = \dot{m}_{belt,2} - \dot{m}_{belt,3}$$

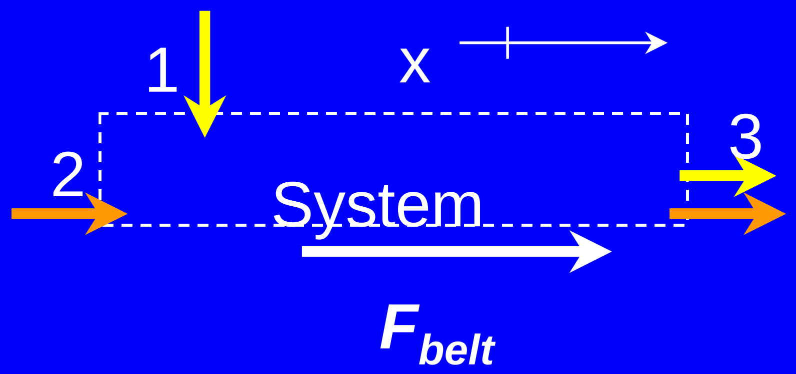
0, SS

$$\frac{dm_{grain,sys}}{dt} = \dot{m}_{grain,1} - \dot{m}_{grain,3}$$

$$\dot{m}_{belt,2} = \dot{m}_{belt,3}$$

$$\dot{m}_{grain,1} = \dot{m}_{grain,3}$$

Conservation of Linear Momentum for this system. (X-direction)



$$\cancelto{0,\ SS}{\frac{dP_{x,sys}}{dt}} = \sum_j F_{x,ext,j} + \sum_{in} \dot{m}_i V_{x,i} - \sum_{out} \dot{m}_e V_{x,e}$$

$$0 = F_{belt} + \dot{m}_{grain,1} V_{x,grain,1} + \dot{m}_{belt,2} V_{x,belt,2} - \dot{m}_{belt,3} V_{x,belt,3} - \dot{m}_{grain,} V_{x,grain,3}$$

$$0 = F_{belt} + \dot{m}_{grain}\left(\cancelto{0}{V_{x,grain,1}} - V_{x,grain,3}\right) + \dot{m}_{belt}\left(\cancelto{0}{V_{x,belt,2} - V_{x,belt,3}}\right)$$

$$\boxed{F_{belt} = \dot{m}_{grain} V_{x,grain,3} = \dot{m}_{grain} V_{belt}}$$

End of Examples

For additional information about the RH Sophomore Engineering Curricula *or* the Systems, Accounting, and Modeling Approach contact ---

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http://www.foundationcoalition.org