Conservation and Accounting Framework — An Introduction

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Introduction

The conservation and accounting framework for engineering science structures topics around several common concepts to help students (1) grasp relationships between apparently disparate concepts and (2) apply a common and powerful problem-solving methodology for a wide range of physical situations. The basic concepts introduced here have been used by the Foundation Coalition to create innovative engineering science courses that cover statics, dynamics, fluid mechanics, thermodynamics, mechanics of materials, circuits, and system dynamics.

System

A **system** is any region in space or quantity of matter set aside for analysis. Everything not inside the system is in the **surroundings**. The system **boundary** is an infinitesimally thin surface, real or imagined, that separates the system from its surroundings.

Systems may be classified according to whether they exchange mass with the surroundings. A **closed system** is a system that does not exchange mass with its surroundings; thus, it has a fixed and unchanging mass although its volume may change. An **open system** is a system that may exchange mass with its surroundings; thus, its mass and volume may change.

Property

A **property** is any characteristic of a system that can be given a numerical value without regard to the history of the system. Properties are classified as either intensive or extensive.

An **intensive property** has a value at a point and its value is independent of the extent or size of the system. The value of an intensive property may vary with time and its position within the system. Examples of intensive properties include temperature, velocity, mass density, specific volume, and specific energy.

An **extensive property** does not have a value at a point and its value depends on the extent or size of the system. The amount of an extensive property for a system can be determined by summing the amount of the extensive property for each subsystem within the system. Examples of extensive properties include mass, charge, linear momentum, volume, and energy.

Accounting Principle

Experience has taught us that the behavior of a system depends upon how certain extensive properties—specifically mass, momentum, charge, energy, and entropy—change with time. We postulate that this change or **accumulation** of an extensive property occurs by one of two mechanisms: (1) **transport** of the extensive property across the system boundary and (2) **generation** (production) or consumption (destruction) of the extensive property within the system. For example, the charge within a system is related to the amount of charge transported across the boundary and the amount of charge generated (or consumed) within the system. This simple balance—**accumulation = transport + generation**— is referred to as the **accounting principle** for an extensive property. Although this principle can be applied to a system for any extensive property, it is especially useful for properties that are conserved.

For a finite-time period, the accounting principle can be written in the **accumulation form** :

$$\begin{bmatrix}\text{Final}\\\text{Amount}\end{bmatrix}-\begin{bmatrix}\text{Initial}\\\text{Amount}\end{bmatrix}=\begin{bmatrix}\text{Amount}\\\text{Entering}\end{bmatrix}-\begin{bmatrix}\text{Amount}\\\text{Leaving}\end{bmatrix}+\begin{bmatrix}\text{Amount}\\\text{Generated}\end{bmatrix}-\begin{bmatrix}\text{Amount}\\\text{Consumed}\end{bmatrix}$$

For a specified time interval, the amount accumulated within the system equals the net amount that entered the system plus the net amount generated within the system.

For an infinitesimal-time period, the accounting principle can be written in the **rate form** :

$$\begin{bmatrix}\text{Rate of}\\\text{Change}\end{bmatrix}=\begin{bmatrix}\text{Transport}\\\text{Rate In}\end{bmatrix}-\begin{bmatrix}\text{Transport}\\\text{Rate Out}\end{bmatrix}+\begin{bmatrix}\text{Generation}\\\text{Rate}\end{bmatrix}-\begin{bmatrix}\text{Consumption}\\\text{Rate}\end{bmatrix}$$

At a specified time, the rate of change within the system equals the net transport rate into the system plus the net generation rate within the system.

Four questions must be answered when writing the accounting principle for a new property: What is the extensive property? How can it be stored within the system? How can it be transported across a system boundary? How can it be generated or consumed inside the system? As shown on the next page, this leads to a consistent way of thinking about and writing the fundamental laws of physical systems.

Conserved Property

Empirical evidence as codified by science has identified a class of extensive properties that can neither be created nor destroyed. An extensive property that satisfies this requirement is called a **conserved property**. When the accounting principle is developed for a conserved property, both the generation rate and the consumption rate are identically zero. We typically consider five conserved properties in engineering science: mass, energy, charge, linear momentum, and angular momentum. Another important property entropy is not conserved. Nonetheless, the accounting framework also provides a useful vehicle for analysis since entropy can only be produced as a consequence of the Second Law of Thermodynamics.

Processes and Interactions

Systems change state by interacting with their surroundings. A **process** is the means by which a system changes state. An **interaction** between a system and its surroundings occurs when an extensive property crosses the system boundary.

Representative Basic Laws

Extensive Property	Rate Form of the Accounting Principle
Generic Property *B* $B_{sys} = \int_{V_{sys}} b r dV$ where $b = \frac{B}{m}$	$\underbrace{\frac{d}{dt}B_{sys}}_{\text{Rate of change of } B \text{ in the system}} = \underbrace{\left[\dot{B}_{\text{transport,in}} - \dot{B}_{\text{transport,out}}\right]}_{\text{Net transport rate of } B \text{ into the system}} + \underbrace{\left[\dot{B}_{\text{generation}} - \dot{B}_{\text{consumption}}\right]}_{\text{Net generation rate of } B \text{ inside the system}}$ Net transport rate of B into the system $\rightarrow$ $\underbrace{\left[\dot{B}_{\text{transport,in}} - \dot{B}_{\text{transport,out}}\right]_{\text{non-flow boundary}} + \left[\sum_{\text{in}} \dot{m}_i b_i - \sum_{\text{out}} \dot{m}_e b_e\right]_{\text{flow boundary}}}_{\text{Transport terms can be separated into two groups}}$ Transport terms can be separated into two groups: transports that occur without mass flow and transports that occur with mass flow
Mass $m_{sys} = \int_{V_{sys}} r dV$	$\underbrace{\frac{d}{dt}m_{sys}}_{\text{Rate of change of mass in the system}} = \underbrace{\sum_{\text{in}} \dot{m}_i - \sum_{\text{out}} \dot{m}_e}_{\text{Net transport rate of mass into the system with mass flow}}$
Linear Momentum $\mathbf{P}_{sys} = \int_{V_{sys}} \mathbf{V} r dV$	$\underbrace{\frac{d}{dt}\mathbf{P}_{sys}}_{\text{Rate of change of linear momentum in the system}} = \underbrace{\sum_j \mathbf{F}_j}_{\text{Net transport rate of linear momentum into the system by external forces}} + \underbrace{\left[\sum_{\text{in}} \dot{m}_i \mathbf{V}_i - \sum_{\text{out}} \dot{m}_e \mathbf{V}_e\right]}_{\text{Net tranport rate of linear momentum into the system with mass flow}}$
Entropy $S_{sys} = \int_{V_{sys}} s r dV$	$\underbrace{\frac{d}{dt}S_{sys}}_{\text{Rate of change of entropy in the system}} = \underbrace{\sum_j \frac{\dot{Q}_j}{T_{b,j}}}_{\text{(a)}} + \underbrace{\left[\sum_{\text{in}} \dot{m}_i s_i - \sum_{\text{out}} \dot{m}_e s_e\right]}_{\text{Net transport rate of entropy into the system with mass flow}} + \underbrace{\dot{S}_{\text{gen}}}_{\text{(b)}}$ (a) Net transport rate of entropy into the system by heat transfer where T_b is the absolute temperature of the boundary j where the heat transfer occurs (b) Entropy generation rate inside the system which is always ≥ 0 except in the limit of an internally reversible process when it equals zero.

General Comments

- Each law has the same format when written using this approach so students begin to see similar terms and physical concepts.
- When solving problems, students are encouraged to write the pertinent basic law and then develop a *problem-specific* model. Thus, students repeatedly see connections back to the basic laws by developing a model instead of memorizing special cases.
- For example, consider how the linear momentum equation could be applied to different problems:
  — For a projectile (a closed system), the $\dot{m}$ terms drop out and Newton's second law is recovered as $m\mathbf{a} = \mathbf{F}_{drag} + m\mathbf{g}$.
  — For a closed system of two cars colliding at an intersection with negligible friction forces, $\mathbf{F}_{surface}$ and the $\dot{m}$ terms drop out; thus, $\mathbf{P}_{final} = (m_A\mathbf{V}_A + m_B\mathbf{V}_B)_{final} = (m_A\mathbf{V}_A + m_B\mathbf{V}_B)_{initial} = \mathbf{P}_{initial}$.
  — For a fluid flowing steadily through a pipe elbow (an open system), $d\mathbf{P}_{sys}/dt = 0$; thus, the surface forces, the system weight, and the mass transport of momentum are related by the equation $0 = \mathbf{F}_{surface} + m_{sys}\mathbf{g} + \dot{m}(\mathbf{V}_{in} - \mathbf{V}_{out})$.

Problem Solving

Given a problem in the basic engineering sciences, the following questions help students formulate a solution:

- What exactly are we trying to find? What is known from the problem statement?
- What is the system? (This must be explicit to apply the accounting principles because they are written for a system.)
- What are the important extensive properties to count? (This encourages students to think in terms of the quantities found in the physical laws.)
- What's the time interval—finite time, transient, or steady state?
- How does the system interact with its surroundings? (Interactions depend on the boundary selected and the properties counted.)
- What are the modeling assumptions that can help us simplify the basic equations for this specific problem?
- How many equations are required to solve for the unknowns?
- If the accounting equations are insufficient, what other equations (constitutive equations) may be used to relate the unknowns in the problem?

Looking for ways to improve your students' problem solving abilities and see connections between topics? Revising your curriculum? Intrigued by these ideas? The Foundation Coalition would like to help you explore these ideas by sharing our experience in developing courses and curricula based on this approach. For questions or suggestions on where to start, contact

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or the Foundation Coalition web site at http://www.foundationcoalition.org.