

## Conservation of Energy, the Work-Energy Principle, and the Mechanical Energy Balance

In your study of engineering and physics, you will run across a number of engineering concepts related to energy. Three of the most common are Conservation of Energy, the Work-Energy Principle, and the Mechanical Engineering Balance. The Conservation of Energy is treated in this course as one of the overarching and fundamental physical laws. The other two concepts are special cases and only apply under limited conditions. The purpose of this note is to review the pedigree of the Work-Energy Principle, to show how the more general Mechanical Energy Balance is developed from Conservation of Energy, and finally to describe the conditions under which the Mechanical Energy Balance is preferred over the Work-Energy Principle.

### Work-Energy Principle for a Particle

Consider a particle of mass  $m$  and velocity  $\vec{V}_G$  moving in a gravitational field of strength  $\vec{g}$  subject to a surface force  $\vec{R}_{surface}$ . Under these conditions, writing Conservation of Linear Momentum for the particle gives the following:

$$\frac{d}{dt}(m\vec{V}_G) = \vec{R}_{surface} + m\vec{g} \quad (1.1)$$

Forming the dot product of Eq. (1.1) with the velocity of the particle and rearranging terms gives the **rate form of the Work-Energy Principle for a particle**:

$$\frac{d}{dt} \underbrace{\left( m \frac{V^2}{2} \right)}_{\text{Kinetic energy}} + \frac{d}{dt} \underbrace{(mgz)}_{\text{Gravitational potential energy}} = \underbrace{\vec{R}_{surface} \cdot \vec{V}_G}_{\text{mechanical power into the system}} \Rightarrow \frac{d}{dt} (E_K + E_{GP}) = \dot{W}_{mech,in} \quad (1.2)$$

Recall that mechanical power is defined as  $\dot{W}_{mech,in} = \vec{R}_{surface} \cdot \vec{V}_G$ , the dot product of the surface force with the velocity of its point of application. Because a particle has no extent and only one velocity, the point of application of the surface force and the particle always have the same velocity.

Integrating Eq. (1.1) with respect to distance or Eq. (1.2) with respect to time gives the more familiar relation for change in kinetic energy, change in gravitational potential energy, and mechanical work:

$$\Delta \underbrace{\left( m \frac{V_G^2}{2} \right)}_{=E_K} + \Delta \underbrace{(mgz_G)}_{=E_{GP}} = \int_{t_1}^{t_2} \underbrace{\vec{R}_{surface} \cdot \vec{V}_G}_{=d\bar{s}} dt = \int_1^2 \underbrace{\vec{R}_{surface} \cdot d\bar{s}}_{=dW_{mech,in}} = W_{mech,in} \quad (1.3)$$

$$\Downarrow$$

$$\Delta E_K + \Delta E_{GP} = W_{mech,in}$$

This is known as the **finite-time form of the Work-Energy Principle for a particle**. Recall that mechanical work is the time integral of the mechanical power and can be calculated as the dot product of the surface force and the displacement of its point of application. Again, the dis-

placement of the point of application of the surface force is unambiguous for a particle because it is the same as the displacement of the particle.

Although the Work-Energy Principle uses energy language — energy, work, power — it adds nothing new that could not have been discovered through a careful application of the Conservation of Linear Momentum.

### Conservation of Energy and the Mechanical Energy Balance for a Closed System

Writing Conservation of Energy for a closed system, we obtain the **rate form of Conservation of Energy for a closed system**:

$$\frac{d}{dt}(E_{sys}) = \dot{Q}_{net,in} + \dot{W}_{net,in} \quad (1.4)$$

Restricting ourselves to only three types of energy — internal energy  $U$ , kinetic energy  $E_k$ , and gravitational potential energy  $E_{GP}$  — we have the following result:

$$\frac{d}{dt}(U_{sys} + E_{K,sys} + E_{GP,sys}) = \dot{Q}_{net,in} + \dot{W}_{net,in} \quad (1.5)$$

Although the distinctions are somewhat artificial, we will segregate the energy into two groups: thermal energy and mechanical energy. Internal energy is usually classified as *thermal energy* because changing internal energy of a system is often associated with a change in temperature. The other two energies are classified as *mechanical energy* because changing the kinetic energy or gravitational potential energy of a system can be done solely by the application of a surface force and its associated mechanical work. In addition, we will separate the work transfer rate of energy (power) into two terms: mechanical work where there is an identifiable surface force and non-mechanical work, e.g. electrical work.

Using these new distinctions between mechanical and thermal phenomena, we can rewrite Eq. (1.5) and group the mechanical and thermal terms as shown below:

$$\begin{aligned} \frac{d}{dt}(U_{sys} + E_{K,sys} + E_{GP,sys}) &= \dot{Q}_{net,in} + \underbrace{\dot{W}_{net,mech,in} + \dot{W}_{net,nonmech,in}}_{=\dot{W}_{net,in}} \\ \underbrace{\frac{d}{dt}(E_{K,sys} + E_{GP,sys})}_{\substack{\text{Rate of change} \\ \text{of the} \\ \text{mechanical energy} \\ \text{in the system}}} &= \underbrace{\dot{W}_{net,mech,in}}_{\substack{\text{Transport rate} \\ \text{of energy by} \\ \text{mechanical work}}} + \underbrace{\left[ \dot{Q}_{net,in} + \dot{W}_{net,nonmech,in} - \frac{d}{dt}(U_{sys}) \right]}_{\substack{\text{Net production rate} \\ \text{of mechanical energy} \\ \text{inside the system}}} \end{aligned} \quad (1.6)$$

$$\Downarrow$$

$$\frac{d}{dt}(E_{K,sys} + E_{GP,sys}) = \dot{W}_{net,mech,in} + \dot{E}_{net,mech,prod}$$

This is called the **rate form of the Mechanical Energy Balance for a closed system**. It accounts for the storage, transport, and production or destruction of mechanical energy in a closed system. In words,

The time-rate-of-change of the mechanical energy in the system equals the net transport rate of energy with mechanical work (net mechanical power) into the system plus the net production rate of mechanical energy inside the system.

In general, the net production rate term can take on both positive and negative values.

The introduction and presence of a production term does not violate Conservation of Energy because we are only counting one type of energy, and one of the characteristics of energy is that it can be stored in different ways. [Consider a marble rolling up and down the sides of a wooden salad bowl. If there are no losses, there is a continuous interchange between kinetic and gravitational potential energy and if one was only counting kinetic energy it would alternately appear to be produced and then destroyed.] The idea of counting only one type of energy is analogous to the idea of counting only a single chemical species (Species Accounting) used earlier in our study of Conservation of Mass.

### **Mechanical Energy Balance = Work-Energy Principle?**

We've already shown that the Work-Energy Principle for particle is a direct descendant of Conservation of Linear Momentum and the Mechanical Energy Balance for a closed system grew out of the Conservation of Energy. Rewritten below together, we see that they appear to be similar even though one is written for a particle and the other for a more general closed system:

*Work-Energy Principle for a particle*

$$\frac{d}{dt}(E_K + E_{GP}) = \dot{W}_{mech,in} \quad (1.2)$$

*Mechanical Energy Balance for a closed system*

$$\frac{d}{dt}(E_{K,sys} + E_{GP,sys}) = \dot{W}_{net,mech,in} + \dot{E}_{net,mech,prod} \quad (1.6)$$

The only significant difference between the two equations is the net production rate term for mechanical energy.

If we can find a set of conditions under which mechanical energy is neither produced nor destroyed, the Work-Energy Principle and the Mechanical Energy Balance contain the same information. So the important question is when does this occur? Without creating an inclusive list of conditions, we will state only one set of conditions:

***Mechanical energy will be neither produced nor destroyed within a closed system if (1) the materials in the system are incompressible, (2) there is no internal friction or friction between parts of the closed system, and (3) only mechanical work occurs on the system boundary. (Note that this does not prohibit friction on the boundary of the system.)***

This set of conditions is consistent with the conditions usually invoked when applying the Work-Energy Principle. When these conditions are satisfied, the Mechanical Energy Balance reproduces the results of the Work-Energy Principle with the added advantage that it applies to any closed system. If there is the restriction on internal friction is relaxed, we will show later that mechanical energy can only be destroyed. With this in mind, conditions that do not produce or destroy mechanical energy frequently represent the *best* or *ideal* behavior for the system. (This idea will be explored further when we encounter the Second Law of Thermodynamics and the Entropy Accounting Principle.)

### Adding Springs (Elastic Energy) to the Mechanical Energy Balance for a closed system

Now that we've shown the relationship between the Work-Energy Principle and the Mechanical Energy Balance, we wish to include an additional type of energy that can be handled within our Mechanical Energy Balance — elastic or spring energy.

Typically we will only consider true mechanical springs. Assuming a linear spring with no hysteresis or internal friction, the energy stored in a spring can be calculated from the following equation:

$$E_{Spring} = \frac{1}{2}k(x - x_o)^2$$

where  $k$  = spring constant [Force/Length] (1.7)  
 $x_o$  = unstretched length of the spring  
 $x$  = stretched length of the spring

Note that an unstretched spring stores no mechanical energy, and that a linear, ideal spring stores energy when it compressed or stretched.

Including this additional form of mechanical energy, we have an **expanded rate form of the Mechanical Energy Balance for a closed system**:

$$\frac{d}{dt}(E_{K,sys} + E_{GP,sys} + E_{Spring,sys}) = \dot{W}_{net,mech,in} + \dot{E}_{net,mech,prod} \quad (1.8)$$

This is the most general form we will present.

### Bottom Line — When should and can I use the Mechanical Energy Balance?

Although the general form can be useful, Eq. (1.8) is most useful when we can assume that there is no mechanical energy production or destruction. Under these conditions, we have the **expanded rate form of the Mechanical Energy Balance for a closed, incompressible system with no internal friction and only mechanical work**:

$$\frac{d}{dt}(E_{K,sys} + E_{GP,sys} + E_{Spring,sys}) = \dot{W}_{net,mech,in} \quad (1.9)$$

Integrated with respect to time we recover the **finite-time form**:

$$\Delta E_{K,sys} + \Delta E_{GP,sys} + \Delta E_{Spring,sys} = W_{net,mech,in} \quad (1.10)$$

If your system contains only incompressible objects, there is no internal friction, and only mechanical work occurs on the boundary of the system then you can and should use Eq. (1.9) or (1.10) instead of the complete Conservation of Energy. This will also replace the Work-Energy Principle for a particle, unless you find an particular advantage in starting with the Conservation of Linear Momentum and integrating. When applied appropriately and correctly, the Mechanical Energy Balance as presented in Eq. (1.9) and Eq. (1.10) can do everything the Work-Energy Principle can and *more*.