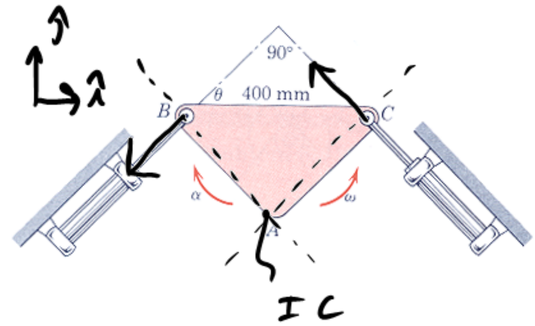
Example Problem - Le 20

5.136 At the instant shown, θ=45 deg and the plate ABC has a counterclockwise angular velocity of 20 rad/s and a clockwise angular acceleration of 100 rad/s^2.
Determine:

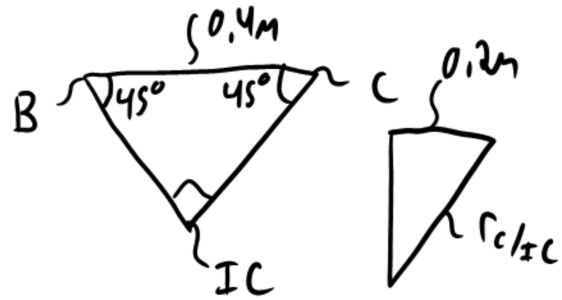
a) the magnitude of the velocity of the piston attached to C,
b) the magnitude of the acceleration of the piston attached to C

(*taken from Dynamics, 3rd Edition by Merriam & Kraige*)



a) Kinematics of ABC

$$v_C = \omega\, r_{C/IC}$$
$$= \left(20\tfrac{rad}{s}\right)\left(r_{C/IC}\right)$$



$$r_{C/IC} = \frac{(0.2m)}{\cos 45°}$$

Geometry/Trig:

$$r_{C/IC} = \frac{0.2m}{\cos 45°}$$

we get

$$\boxed{v_C = 5.657\,m/s}$$

Find velocity

$$\vec{v}_C = v_C\left(-\cos 45°\,\hat{\imath} + \sin 45°\,\hat{\jmath}\right)$$
$$= -4\,m/s\,\hat{\imath} + 4\,m/s\,\hat{\jmath}$$

b)
$$\vec{a}_C = \vec{a}_B + \vec{\alpha}\times\vec{r}_{C/B} - \omega^2\vec{r}_{C/B}$$

$$\vec{\alpha} = \left(-100\tfrac{rad}{s^2}\hat{k}\right)$$

$$\vec{r}_{C/B} = \left(0.4m\,\hat{\imath}\right)$$

$$\vec{a}_C = \vec{a}_B - 40\,m/s^2\,\hat{j} - 160\,m/s^2\,\hat{i}$$

write $\vec{a}_C = a_{Cx}\hat{i} + a_{Cy}\hat{j}$

$\vec{a}_B = a_{Bx}\hat{i} + a_{By}\hat{j}$

$\hat{i})\quad a_{Cx} = a_{Bx} - 160 \quad (1)$

$\hat{j})\quad a_{Cy} = a_{By} - 40 \quad (2)$

Constraints on B + C

$\vec{a}_B = a_B(\cos 45°\,\hat{i} + \sin 45°\,\hat{j})$

$\vec{a}_C = a_C(-\cos 45°\,\hat{i} + \sin 45°\,\hat{j})$

$a_{Bx} = a_B \cos 45° \quad (3)$

$a_{By} = a_B \sin 45° \quad (4)$

$a_{Cx} = -a_C \cos 45° \quad (5)$

$a_{Cy} = a_C \sin 45° \quad (6)$

solve for $a_C = 84.85\,m/s^2$