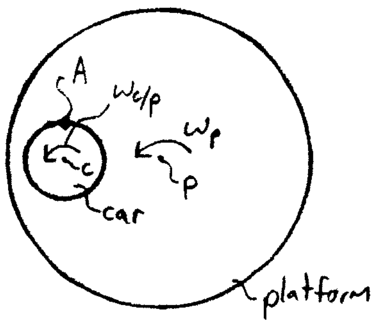
Example : Carnival Ride



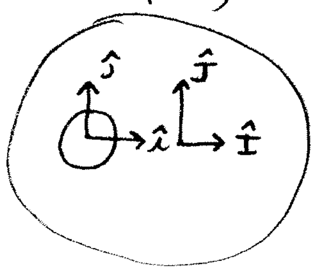
The platform has constant angular velocity $\omega_p = 1.5 \ \text{rad/s}$ ccw
The car has constant ang. vel <u>relative to the platform</u> of $\omega_{c/p} = 2 \ \text{rad/s}$ ccw.

At the instant shown <u>find vel and accel</u> of the rider sitting at point A.

dist from c to A is 0.75m
dist from c to p is 3.0m

Strategy: no masses, no forces $\Rightarrow$ try kinematics first

Moving axis at pt C, attached to car Fixed axis at P, attached to ground



note: $\hat{i} = \hat{I}$
$\hat{j} = \hat{J}$

a) $\vec{v}_A = \vec{v}_o + \vec{v}_{rel} + \vec{\omega} \times \vec{r}_{A/o}$

$\vec{v}_o$ = vel of pt C wrt ground $= \vec{\omega}_p \times \vec{r}_{c/p} = (1.5 \ \text{rad/s} \ \hat{K}) \times (-3\text{m} \ \hat{I})$
$= -4.5 \ \text{m/s} \ \hat{J}$

$\vec{v}_{rel} = \vec{0}$ since moving coord is attached to the car

$\vec{\omega} = (1.5 + 2) \ \text{rad/s} \ \hat{K}$ spin of platform adds to spin of car

$\vec{r}_{A/o} = 0.75\text{m} \ \hat{j} = 0.75\text{m} \ \hat{J}$

so $\vec{v}_A = (-4.5 \ \text{m/s} \ \hat{J}) + \vec{0} + (3.5 \ \text{rad/s} \ \hat{K}) \times (0.75\text{m} \ \hat{J})$

$$\boxed{\vec{v}_A = -2.625 \ \text{m/s} \ \hat{I} - 4.5 \ \text{m/s} \ \hat{J}}$$

b) $\vec{a}_A = \vec{a}_o + \vec{a}_{rel} + \vec{\alpha} \times \vec{r}_{A/o} - \omega^2 \vec{r}_{A/o} + 2\vec{\omega} \times \vec{v}_{rel}$

$\vec{a}_o$ = accel of pt C wrt ground $= \underbrace{\vec{\alpha}_p}_{0 \text{ constant } \omega_p} \times \vec{r}_{c/p} - \omega_p^2 \vec{r}_{c/p}$

$= -(1.5 \text{ rad/s})^2 (-3\text{m}\,\hat{I})$

$= 6.75 \text{ m/s}^2\,\hat{I}$

$\vec{a}_{rel} = \vec{0}$ moving coord fixed to car

$\vec{\alpha} = (\vec{\alpha}_{c/p} + \vec{\alpha}_p) = \vec{0}$ all constant ang vel

so $\vec{a}_A = (6.75 \text{ m/s}^2\,\hat{I}) - (3.5 \text{ rad/s})^2 (0.75\text{m}\,\hat{J}) + 2(3.5 \text{ rad/s}\,\hat{K}) \times (\vec{0})$

$$\boxed{\vec{a}_A = 6.75 \text{ m/s}^2\,\hat{I} - 9.1875 \text{ m/s}^2\,\hat{J}}$$

This is not the only way to solve this problem. If you placed the moving coord system at pt P, but attached it to the platform,

$\vec{v}_A = \vec{v}_o + \vec{v}_{rel} + \vec{\omega} \times \vec{r}_{A/o}$

$\vec{v}_o = \vec{0}$

$\vec{v}_{rel} = (2 \text{ rad/s}\,\hat{k}) \times (0.75\hat{j}) = -1.5 \text{ m/s}\,\hat{i} = -1.5 \text{ m/s}\,\hat{I}$

$\vec{\omega} = (1.5 \text{ rad/s}\,\hat{K})$

$\vec{r}_{A/o} = (-3\text{m}\,\hat{i} + 0.75\text{m}\,\hat{j}) = -3\text{m}\,\hat{I} + 0.75\text{m}\,\hat{J}$

$\vec{a}_A = \vec{a}_o + \vec{a}_{rel} + \vec{\alpha} \times \vec{r}_{A/o} - \omega^2 \vec{r}_{A/o} + 2\vec{\omega} \times \vec{v}_{rel}$

$\vec{a}_o = \vec{0}$

$\vec{a}_{rel} = -(2 \text{ rad/s})^2 (0.75\text{m}\,\hat{j}) = -3 \text{ m/s}\,\hat{j} = -3 \text{ m/s}\,\hat{J}$

$\vec{\alpha} = \vec{0}$

Yields the <u>same eqns</u> for $\vec{v}_A$ + $\vec{a}_A$

Some terms are more difficult, some are easier. Such is life.

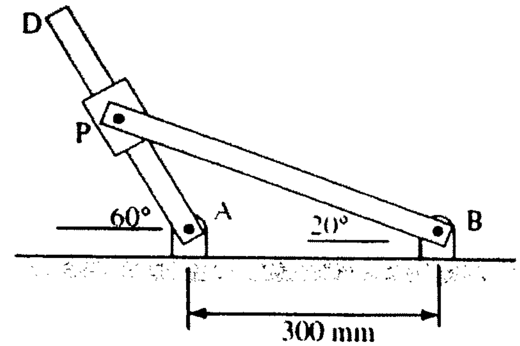
Example

Two rotating rods are connected by slider block P. Neglect friction between the slider block and bar AD. The rod attached at A rotates with a constant angular velocity of 10 rad/s clockwise. Determine:

a) the angular acceleration of the rod attached at B,

b) the relative acceleration of slider block P with respect to the rod on which it slides.

c) If bar BP has a mass of 5 kg the mass of the slider is negligible, determine the reactions at point B.



Strategy : a) Kinematics
b) Kinematics
c) CAMrate, CLMrate

Use values from HW problem 4.34 : $\vec{\omega}_{BP} = -5.17 \text{ rad/s } \hat{K}$

$v_{rel} = 1.344 \text{ m/s}$ (along bar A → D)

$\vec{\omega}_{AD} = -10 \text{ rad/s } \hat{K}$

a) Find $\vec{\alpha}_{BP}$

Kinematics of bar BP, accel

$$\vec{a}_P = \vec{a}_B + \vec{\alpha}_{BP} \times \vec{r}_{P/B} - \omega_{BP}^2 \, \vec{r}_{P/B}$$

$$\vec{a}_P = a_{Px}\hat{I} + a_{Py}\hat{J}$$

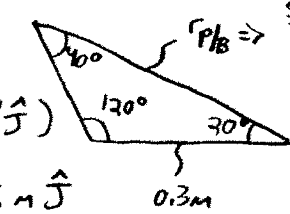
$$\vec{a}_B = \vec{0}$$

$$\vec{\alpha}_{BP} = \alpha_{BP}\hat{K}$$

$\vec{r}_{P/B}$ = use law of sines

$= (0.404\text{m})(-\cos 20°\hat{I} + \sin 20°\hat{J})$

$= -0.380\text{m}\hat{I} + 0.138\text{m}\hat{J}$



$r_{P/B} \Rightarrow \dfrac{\sin 120°}{r_{P/B}} = \dfrac{\sin 40°}{0.3\text{m}}$

$r_{P/B} = 0.404\text{m}$

$$\omega_{BP} = -5.17 \text{ rad/s}$$

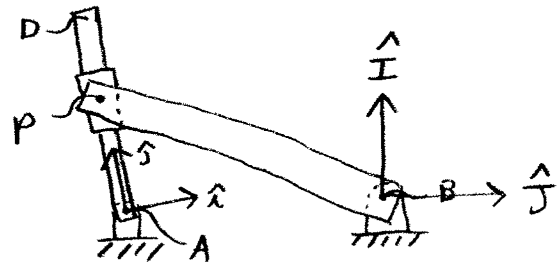
yields $a_{Px}\hat{I} + a_{Py}\hat{J} = (\alpha_{BP}\hat{K}) \times (-0.380\text{m}\hat{I} + 0.138\text{m}\hat{J}) - (-5.17 \tfrac{\text{rad}}{\text{s}})^2(-0.380\text{m}\hat{I} + 0.138\text{m}\hat{J})$

$\hat{I})\quad a_{px} = -\alpha_{BP}(0.138\,m) + 10.16\,m/s^2 \qquad (1)$

$\hat{J})\quad a_{py} = -\alpha_{BP}(0.380\,m) - 3.69\,m/s^2 \qquad (2)$

Now that we know something about $\vec{a}_P$, go to the moving axis relation

Define coord systems



fixed coord in usual direction

moving coord centered at A, with $\hat{\jmath}$ aligned with A→D

Thus, $\hat{\imath} = \cos 30°\,\hat{I} + \sin 30°\,\hat{J}$
$\hat{\jmath} = -\sin 30°\,\hat{I} + \cos 30°\,\hat{J}$

The accel relation is

$$\vec{a}_P = \vec{a}_o + \vec{a}_{rel} + \vec{\alpha}\times\vec{r}_{P/o} - \omega^2\vec{r}_{P/o} + 2\vec{\omega}\times\vec{v}_{rel}$$

$\vec{a}_P = a_{px}\hat{I} + a_{py}\hat{J}$

$\vec{a}_o = \vec{0}$ (pt A is fixed)

$\vec{a}_{rel} = a_{rel}\,\hat{\jmath} = (a_{rel})(-\sin 30°\,\hat{I} + \cos 30°\,\hat{J}) = a_{rel}(-0.5\,\hat{I} + 0.866\,\hat{J})$

$\vec{\alpha} = \vec{0}$ (constant ang vel)

$\vec{r}_{P/o} = \vec{r}_{P/A}$ = by law of sines, $r_{P/A} = \frac{\sin 20°}{\sin 40°}(0.3\,m) = 0.1596\,m$

$= 0.1596\,m\,\hat{\jmath} = (0.1596\,m)(-\sin 30°\,\hat{I} + \cos 30°\,\hat{J})$

$= -0.0798\,m\,\hat{I} + 0.1382\,m\,\hat{J}$

$\vec{v}_{rel} = 1.344\,m/s\,\hat{\jmath}$ (from HW 4.31)

$= (1.344\,m/s)(-\sin 30°\,\hat{I} + \cos 30°\,\hat{J}) = -0.672\,m/s\,\hat{I} + 1.164\,m/s\,\hat{J}$

$\vec{\omega} = -10\,rad/s\,\hat{K}$

Put it all together using

$\vec{\alpha}\times\vec{r}_{P/o} = \vec{0}$

$-\omega^2\vec{r}_{P/o} = -(-10\,rad/s)^2(-0.0798\,m\,\hat{I} + 0.1382\,m\,\hat{J}) = 7.98\,m/s^2\,\hat{I} - 13.82\,m/s^2\,\hat{J}$

$$2\vec{\omega}\times\vec{v}_{rel} = 2(-10\,^{rad}/_{s}\,\hat{K})\times(-0.672\,^{m}/_{s}\,\hat{I} + 1.164\,^{m}/_{s}\,\hat{J})$$
$$= 23.28\,^{m}/_{s^2}\,\hat{I} + 13.44\,^{m}/_{s^2}\,\hat{J}$$

Yields

$\hat{I})$ $a_{px} = -0.5\,a_{rel} + 7.98\,^{m}/_{s^2} + 23.28\,^{m}/_{s^2}$

$\hat{J})$ $a_{py} = 0.866\,a_{rel} - 13.82\,^{m}/_{s^2} + 13.44\,^{m}/_{s^2}$

simplify

$\hat{I})$ $a_{px} = (-\tfrac{1}{2})a_{rel} + 31.26\,^{m}/_{s^2}$ (3)

$\hat{J})$ $a_{py} = (0.866)a_{rel} - 0.38\,^{m}/_{s^2}$ (4)

Our 4 equations have unknowns a_{px}, a_{py}, α_{BP}, a_{rel} so we can solve

$a_{px} =$

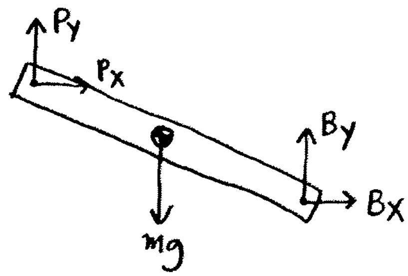
$a_{py} =$

$\alpha_{BP} =$ ang accel of rod attached at pt B (part a)

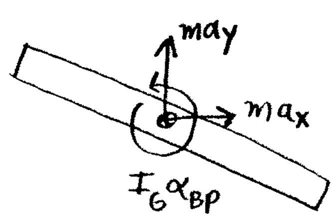
$a_{rel} =$ rel accel of slider block wrt rod AD (part b)

To find part (c) we must use the rate form equations on bar BP

FBD



KD



constraint at P: $P_x = P\sin 60^\circ$, $P_y = P\cos 60^\circ$ } reaction must be perpendicular to rod AD

FAR, so $\vec{a}_G = \vec{\alpha}_{BP}\times\vec{r}_{G/B} - \omega_{BP}^2\,\vec{r}_{G/B}$

↳ 2 eqns, unknowns are a_x, a_y

$CLM_{x\,rate}$, $CLM_{y\,rate}$, $CAM_{G\,rate}$ ⇒ 3 eqns, unknowns are $B_x, B_y, P_x, P_y, a_x, a_y$

add in constraint eqns at P, 7 eqns for $a_x, a_y, \underbrace{B_x, B_y,}_{\text{answer is this}} P_x, P_y, P$