

Exam 2

Name \_\_\_\_\_

Section \_\_\_\_\_

CM# \_\_\_\_\_

Email \_\_\_\_\_ @Rose-Hulman.edu

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Separate your code in three m-files. Your top level code must be called (all lower case)

lastname\_firstname.m

while the functions must be called

lastname\_firstname\_euler.m

lastname\_firstname\_exact.m

Include your name, section number, and CM number in the comments in the header of each file. There should be no output other than what is asked for.

When you *complete* your program and function, *copy and paste* them into the correct directory:  
  
T:\me\ME123\Exam02\Section01 (T is the ``class'' directory.)

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Suppose that we take a bar (helicopter blade) and spin it around with a motor. At first, the blade is stationary, and as we apply a motor torque the blade travels faster and faster. The motor needs to overcome the inertia of the blade, and the drag due to the air. When all of this information is collected in a differential equation for the angular velocity of the blade, we have

$$\frac{d\omega}{dt} = \frac{T_m}{I} - \frac{k}{I}\omega^2 \quad \omega(0) = 0$$

where

- $\omega$  = Angular velocity of blade (rad/s)
- $T_m$  = Motor torque = 1000 N-m
- $I$  = Rotational inertial of blade = 15 kg-m
- $k$  = Net air drag coefficient = 0.1 N-m

- (a) (10 pts) In the space below, write the Euler formula that you could use to solve this problem. For full credit, use “ $n$ ” as the index and  $\Delta t$  as your interval. Express your answer in the form  $\omega_{n+1} = \dots$
- (b) (60 pts) Create a function to implement the Euler formula using an interval of  $\Delta t=0.01$  seconds, for  $\omega$  from 0 to  $0.99\sqrt{\frac{T_m}{k}}$  rad/s. (This is 99% of the steady-state value). The inputs to the function must be  $T_m$ ,  $k$ ,  $I$ , and  $\Delta t$ . The outputs must be the vectors  $t$  and  $\omega$ . Create a top level program which calls the Euler function and plots  $\omega$  as a function of  $t$ . Use a solid line for the curve and label your plot nicely.
- (c) (30 pts) Without air drag, the blade will continue to accelerate indefinitely, and the equation for the angular velocity in this case becomes:

$$\omega_{nodrag} = \frac{T_m}{I} t$$

Write a function to create this exact solution, using an interval of  $\Delta t=0.001$  seconds, for  $\omega$  from 0 to  $\sqrt{\frac{T_m}{k}}$  rad/s. The inputs to this function should be  $T_m$ ,  $k$ ,  $I$ , and  $\Delta t$ . The outputs must be vectors for time and angular velocity. Add this line to your existing plot with a dashed line, and add a legend to your plot to indicate which solution is Euler and which is exact without drag.