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Compression Loading Applied to Round Double Beam Fracture Specimens. I: Application to Materials with Large Characteristic Lengths

ABSTRACT: The round double beam loaded in tension, RDB(T), or wedging, RDB(W), is part of standard test methods for measuring the fracture toughness of aluminum (ASTM E 1307), cemented carbides (ASTM B 644), and rock. When the combination RDB(T) or RDB(W) is used with materials with a relatively large characteristic length, which is defined as the square of the ratio of fracture toughness to tensile strength, the crack tends to depart from the intended fracture plane rendering the data unusable. Loading the RDB in eccentric compression, RDB(B), has been investigated as an acceptable loading alternative in such cases. Tests of over 100 RDB(B) specimens made from different concrete mixtures using three specimen sizes all produced usable data. In addition, the fracture toughness values obtained from RDB(B) tests are consistent with values obtained from RDB(W) tests. The test equipment, test procedures, and data reduction method are all described in detail for the RDB(B).

KEYWORDS: fracture toughness testing, round double beam specimen, eccentric compression loading, characteristic length, concrete

Introduction

In the late 1970s, Barker [1–2] developed a new geometry for fracture toughness test specimens, specifically for materials that are difficult to precrack. Precracking is required in some standardized test methods (i.e., ASTM E 399, Standard Test Method for Plane-Strain Fracture Toughness of Metallic Materials). The specimen was originally called the short rod, but current standardized terminology refers to the test specimen as the round double beam, RDB (ASTM E 1823, Standard Terminology Relating to Fatigue and Fracture Testing). The RDB is a cylindrical specimen with a chevron notch (Fig. 1). The RDB offers several advantages over other specimen geometries [3]. Among these advantages, the chevron notch eliminates the need for fatigue precracking because stable crack growth occurs before critical measurements are obtained during a test. Also, the cylindrical shape results in a smaller specimen volume compared to beams with the same crack length (Fig. 2).

The RDB can be loaded in a number of ways. The original loading technique was tension, Fig. 3a, [1]. ASTM International designates this combination of specimen geometry and loading technique as RDB(T) (ASTM E 1823). A similar loading technique is wedging, Fig. 3b, ASTM designation RDB(W). As will be described later in this paper, these combinations of geometry and loading technique often fail to produce useable results for materials with a high ratio of fracture toughness to tensile strength, which can be expressed as the characteristic length. Therefore, an alternative loading technique, eccentric compression, Fig. 3c, has been explored. The ASTM designation for this combination of specimen geometry and loading technique is RDB(B), where (B) stands for

bending since the boundary conditions resemble three-point bending with a very narrow span length.

The RDB(B) combination has been developed for materials with a large characteristic length. The authors use the definition of characteristic length by Hillerborg et al. [4]; characteristic length, l_{ch} is defined as the square of the ratio of fracture toughness, K_{Ic} to tensile strength, σ , (Eq 1).

$$l_{ch} = (K_{Ic}/\sigma)^2 \quad (1)$$

In order to demonstrate the need for a new loading technique to be used with the RDB specimen, laboratory results are presented for tests on concrete with various aggregate sizes using the RDB(W). In many cases, the crack departs from the intended plane, which renders the results unusable. Such behavior is the result of large tensile stresses parallel to the direction of crack propagation, positive T -stress. This behavior can be anticipated for materials with a relatively large l_{ch} [5]. The RDB(B) combination reduces the positive T -stress, which results in the crack remaining in the intended plane. Therefore, the round double beam loaded in eccentric compression can be used when measuring the fracture toughness of materials with a large l_{ch} . If desired, the RDB(B) combination can also be used with materials that do not have a relatively large l_{ch} .

The results of over 100 tests of concrete using the RDB(B) combination are presented to demonstrate that the crack path remains in the intended plane and that the resulting measured fracture toughness is repeatable. Some areas require further investigation. Those areas are described in the conclusion.

Tension or Wedge Loading Applied to Round Double Beam Specimens

The round double beam loaded in tension or wedging is used in standard test methods for measuring the fracture toughness of aluminum (ASTM E 1304, Standard Test Method for Plane-Strain (Chevron-Notch) Fracture Toughness of Metallic Materials), cemented carbides (ASTM B 771, Standard Test Method for Short

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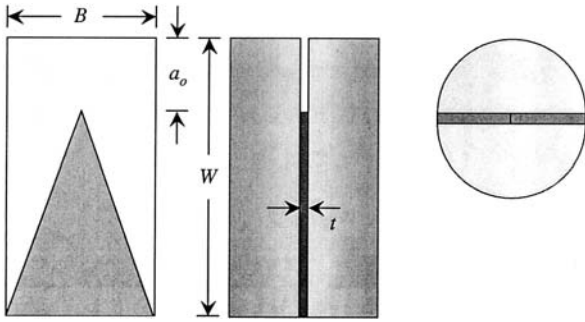


FIG. 1—The round double beam, RDB, specimen for measuring fracture toughness.

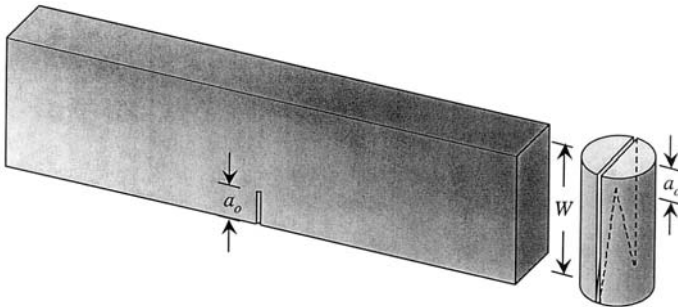


FIG. 2—Comparison of volume of the single edge specimen (left) and round double beam specimen (right) with identical heights and initial cracking lengths.

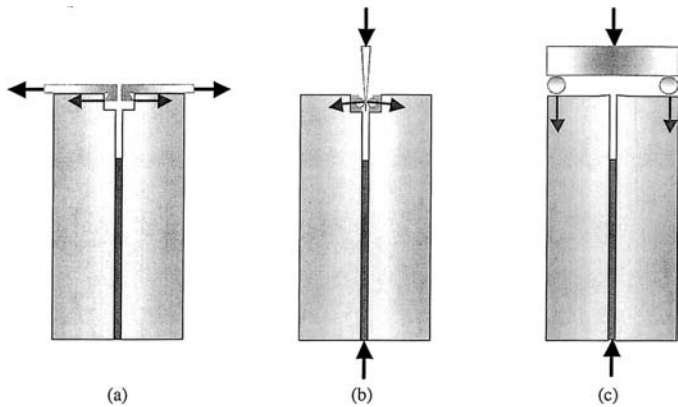


FIG. 3—Loading techniques applied to the round double beam specimen: (a) tension, (b) wedging, and (c) eccentric compression.

Rod Fracture Toughness of Cemented Carbides) and rock [6]. The RDB geometries used in these standards have a length, W , to diameter, B , ratio of 1.45:1 or 2:1. The RDB geometry used in this study has a W/B ratio of approximately 2:1 (Fig. 4). The ratio was chosen because cylinders of concrete with a 2:1 ratio are commonly used for measuring compressive strength (ASTM C 39, Standard Test Method for Compressive Strength of Cylindrical Concrete Specimens), and indirect tensile strength (ASTM C 49b, Standard Test Method for Splitting Tensile Strength of Cylindrical Concrete Specimens) in the United States. The notch geometry was adopted to be similar to the geometry in ASTM E 1304 and to make it easier to cast the notch into specimens made of concrete.

Data reduction is typically applied at two levels. Level I assumes linear elastic fracture mechanics (LEFM) conditions. Level II modifies the Level I result to account for nonlinear conditions.

Data Reduction

The Level I data reduction method was derived by Barker [1]. The chevron notch causes the average stress intensity across a straight crack front, $K_{I,avg}$, to decrease with increasing crack length a up to a critical crack length a_c after which $K_{I,avg}$ increases. Barker showed that under LEFM conditions, the relative critical crack length a_c/B is relatively independent of material and specimen size. This means that $K_{I,avg}$ reaches a minimum at a_c ; therefore, under LEFM conditions, the specimen experiences peak load F_{max} at a_c . The minimum value of $K_{I,avg}$ can be normalized by the specimen size quantities to obtain the normalized minimum stress intensity factor Y_{min}^* , which is dimensionless. The result is that, for Level I data reduction, the apparent fracture toughness K_{IQ} is given by Eq. 2. The value of the normalized minimum stress intensity factor Y_{min}^* depends only upon the geometry of the RDB specimen and the loading technique; therefore, the value of Y_{min}^* is different for an RDB(W) and an RDB(B), even if the specimen geometries are identical. The value of Y_{min}^* for the RDB(B) used in this study is presented in a later section.

$$K_{IQ} = \frac{Y_{min}^*}{B\sqrt{W}} \times F_{max} \tag{2}$$

A Level II data reduction method also was derived by Barker [2]. The method uses information contained in the load versus load line displacement behavior of a test specimen. Therefore, applied load and load line displacement must be measured and recorded throughout the test in order to apply Level II data reduction. In addition, at least one unload-reload cycle must be performed before and after peak load (Fig. 5). Barker [2] named the ratio of the difference in residual displacements $\Delta\delta_o$ to the difference in displacements under load $\Delta\delta$, the inelastic correction factor p (Eq 3). He showed that when limited nonlinear fracture mechanics conditions are present, p can be used to modify the Level I fracture toughness K_{IQ} to obtain the size-independent fracture toughness K_{Ic} (Eq 4).

$$p = \frac{\Delta\delta_o}{\Delta\delta} \tag{3}$$

$$K_{Ic} = K_{IQ} \sqrt{\frac{1+p}{1-p}} \tag{4}$$

Results when Using the RDB(W) with Concrete

A preliminary study was performed on concrete using the RDB(W) combination with relative dimensions as shown in Fig. 4

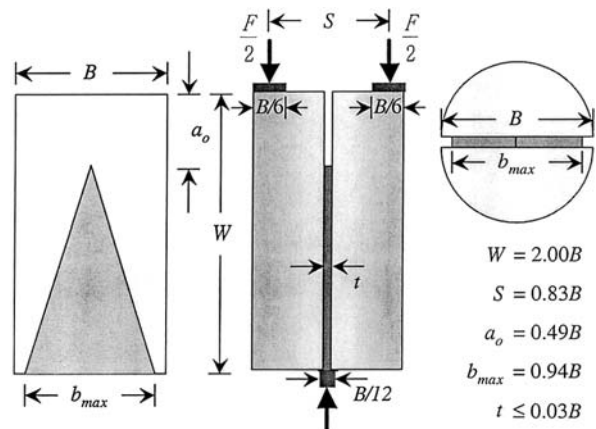


FIG. 4—The round double beam specimen used in this study.

with nominal B_s of 152 mm, 305 mm, and 610 mm. In some cases, the crack remained in the notch plane and usable data were obtained (Fig. 6a). In most cases, the crack deviated from the notch plane rendering the data unusable (Fig. 6b). Another study was undertaken to identify any relation between maximum aggregate size

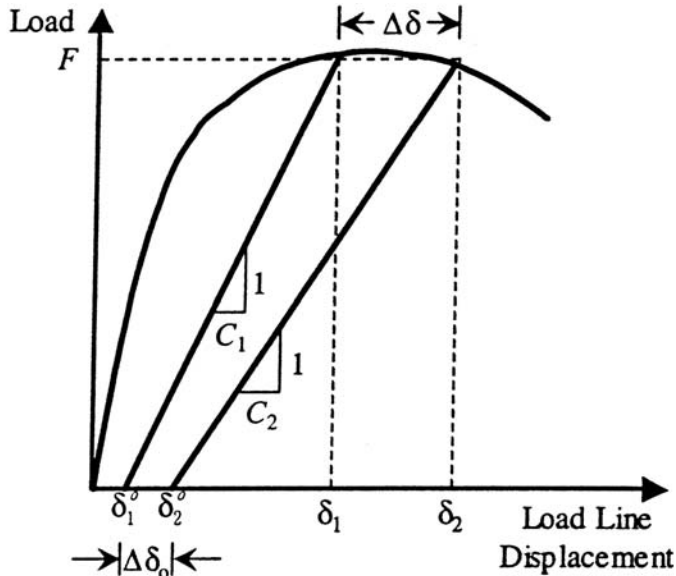


FIG. 5—Example of data obtained in order to perform Level II data reduction.

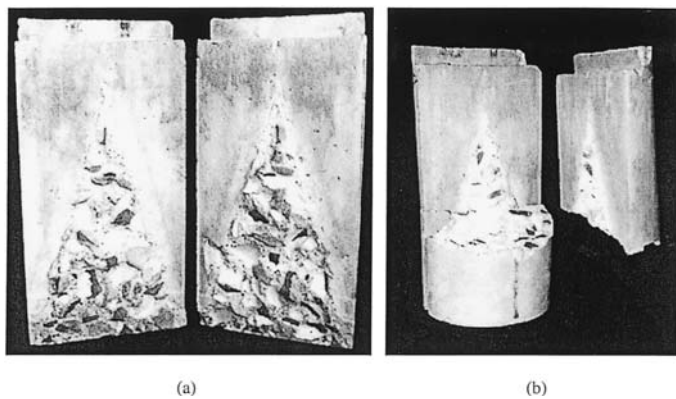


FIG. 6—Tested round double beam specimens under wedge loading: (a) crack remained in the intended plane, (b) crack deviated from the intended plane.

and crack path in the RDB(W). The laboratory test results are summarized in Table 1.

The potential causes of crack deviation are detailed in Hanson and Ingraffea [5] and are summarized here. Wedge or tension loading of an RDB causes large, positive T -stresses. Materials with a relatively large fracture toughness K_{Ic} will require a relatively large applied load F (refer to Eq 2), which will result in a relatively large T -stress. If the tensile strength of the material σ_t is relatively low, the T -stress might exceed σ_t , resulting in a new crack deviating out of the intended plane. Therefore, materials with a relatively large ratio of K_{Ic} to σ_t are likely to exhibit this behavior. The square of this ratio is the characteristic length l_{ch} . Values of K_{Ic} , σ_t , and l_{ch} are summarized in Table 2 for several materials and the concrete used in this study. Other researchers have tested aluminum [7,8] and aluminum oxide [9] using the RDB(T) with a length-to-diameter ratio of 2:1. As expected from review of the l_{ch} values in Table 2, none of those researchers indicated any occurrence of crack deviation.

The crack deviation behavior is not limited, however, to the 2:1 RDB(W) used with concrete. Santos [10] observed the same behavior on an infrequent basis when testing concrete using RDB(T) specimens with a height-to-diameter ratio of 1.45:1. Ingraffea et al. [11] observed crack deviation behavior when testing rock using RDB(T) specimens with a height-to-diameter ratio of 1.5:1.

TABLE 2—Representative characteristic lengths for various materials.

Material	σ_t (MPa)	K_{Ic} (MPa \sqrt{m})	l_{ch} (mm), Eq. 1
Alumina, Al ₂ O ₃ ^a	262	4.0	0.2
Sintered Glass Ceramic ^b	88	1.5	0.3
Polyvinyl Chloride (PVC) ^a	46	2.2	2.7
Styrene Maleic Anhydride (SMA) ^c	31	2.4	6.1
Aluminum, 6061-T651 ^d	320	30	8.8
Steel, AISI 1144 ^a	540	66	14.9
Rock, Dolostone ^e	15.2	1.98	16.9
Concrete, 1 mm	3.62	1.37 ^f	143 ^f
Concrete, 13 mm	3.13	1.84 ^f	343 ^f
Concrete, 25 mm	3.32	1.48 ^f	199 ^f

^a Dowling [13].

^b Boccaccini et al. [14].

^c Chrysostomou and Hashemi [15].

^d Morrison et al. [8].

^e Gunsallus and Kulhawy [16].

^f Based on Level I toughness, K_{Ic} , with $Y_{min}^* = 48.3$.

TABLE 1—Summary of laboratory test results when wedge loading concrete RDB specimens.

	Preliminary Tests				Behavior Study		
	25	25	305	610	152	152	152
Max. aggregate size (mm)	25	25	305	610	152	152	152
Water-cement ratio	0.31	0.58	0.58	0.40	0.40	0.40	0.40
Diameter, B (mm)	152	152	305	610	152	152	152
# usable/# tested	0/5	1/5	0/2	0/1	5/5	3/5	1/10
Avg. K_{Ic} (MPa \sqrt{m}) ^a Max.	...	2.08	1.37	1.84	1.48
Min.	...	2.08	1.53	2.09	1.48
Tensile Strength (MPa)	5.06	3.62	3.13	3.32

^aLevel I toughness (Eq 2) using $Y_{min}^* = 48.3$.

Tschegg et al. [12] report the same behavior when testing asphaltic concrete using wedge loaded, straight notch, cube specimens with a length-to-width ratio larger than 1:1. In order to obtain usable data for these combinations of test specimen geometry and material, something must be changed. One option is to change the loading technique.

Compression Loading Applied to Round Double Beam Specimens

The idea to load the RDB in eccentric compression was proposed by Zehnder [17] as a way to stop the crack deviation behavior. By loading the specimen in this manner, the positive *T*-stress due to bending induced by the eccentric compression is counteracted in part by a negative, compressive, *T*-stress due to the uniform axial stress component induced by the compressive loading (Fig. 7). The result is that none of the RDB(B) specimens tested in this study exhibited crack deviation behavior.

Test Equipment

For Level I data reduction, the testing system must consist of a load frame, actuator, load cell, controller, and data acquisition equipment. In order to obtain sufficient data for Level II reduction, the controller must be closed loop, and two displacement measuring devices must be added. The test must be run in crack mouth opening displacement, CMOD, control. Example capacity requirements for testing a 152 mm diameter RDB(B) made of concrete are peak load of 55 kN, maximum actuator displacement of 6 mm, and maximum CMOD of 1 mm.

To measure CMOD, the devices must be rigidly attached to the specimen. A suggested apparatus (Fig. 8) is based on a similar ap-

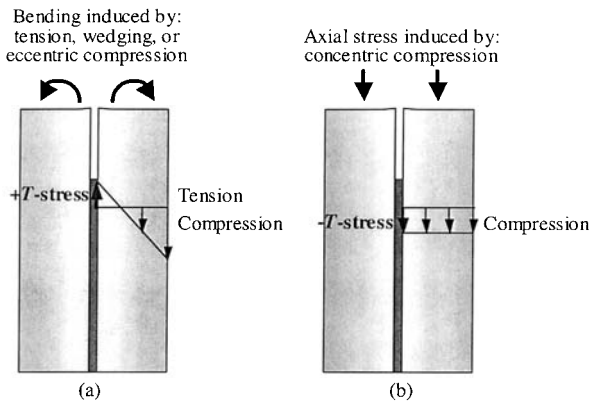


FIG. 7—Schematic of stresses parallel to the direction of crack propagation in an RDB: (a) bending component due to tension, wedge, or eccentric compression loading, (b) axial component due to concentric compression loading.

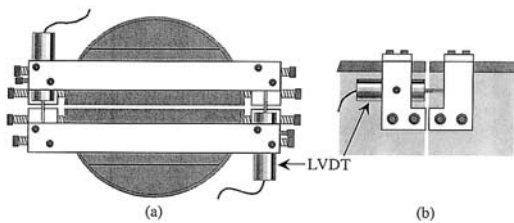


FIG. 8—Suggested apparatus for rigidly attaching displacement measuring devices to the round double beam specimen: (a) top view, (b) side view.

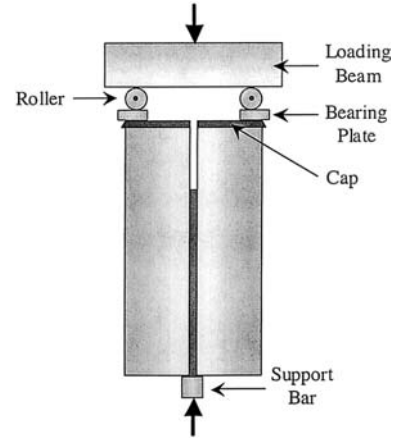


FIG. 9—Loading system for a compression loaded round double beam specimen.

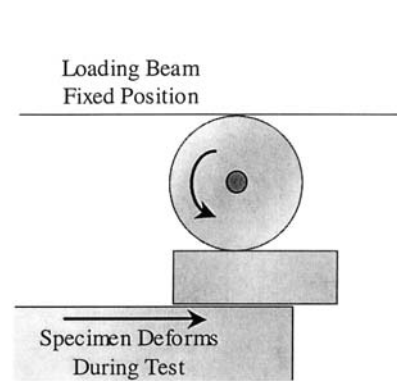


FIG. 10—Schematic of roller and bearing plate contact during an RDB(B) test.

paratus developed by Tschegg [18]. Results from numerical simulations [19] indicate that there is negligible effect on *p* if CMOD is measured within *B*/6 from the top of the specimen.

The loading system is shown schematically in Fig. 9. The actuator loads a beam that should be made of a relatively stiff, strong material such as steel. Based on laboratory results, a steel loading beam with a moment of inertia at least equal to $(B/4)^4$ will be sufficiently stiff. Rollers transfer load from the loading beam to the bearing plates on the specimen. Although the CMOD of the RDB(B) is very small during the test, the rollers must be able to move (Fig. 10). Therefore, the rollers should be of sufficiently large diameter to minimize deformations (*B*/6 or larger is adequate), and of sufficiently strong steel to remain elastic even with the large contact stresses. The bearing plates spread the applied load from the rollers to parts of the top of the specimen. The plates must be sufficiently thick to dissipate the high contact stresses from the roller before they are transferred to the specimen (*B*/12 or thicker is adequate). The top of the specimen must be smooth to avoid stress concentrations beneath the bearing plates. For concrete specimens, a sulfur or gypsum cement cap applied according to ASTM C 617, Standard Practice for Capping Cylindrical Concrete Specimens, can be used. Beneath the specimen is a steel support bar. The bar should be *B*/12 wide and no more than *B*/12 thick in order to provide a stable support during testing.

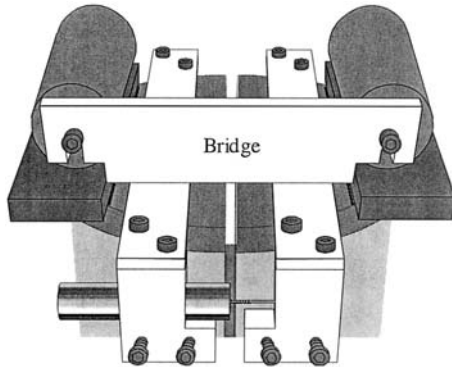


FIG. 11—Schematic of the top of an RDB specimen when ready for compression loading.

Test Procedure

Before testing, all specimen dimensions must be measured. The height W includes any cap used to create a smooth top surface. The centerline should be marked at the base of the specimen to facilitate placement on the support bar. The top of the specimen should be marked to facilitate placement of the bearing plates. The CMOD measuring devices must be attached before placing the loading equipment on the specimen. A bridge can be used to help position the rollers and bearing plates before testing (Fig. 11). Note that the bridge must be removed after a small load (less than 1% of the anticipated peak load is sufficient) is applied through the rollers and bearing plates to the specimen.

The strain rate affects the measured properties of some materials such as concrete. Therefore, the load or CMOD rate should be adjusted so that the peak load is reached within a certain range of time. Based on other proposals for measuring the fracture properties of concrete [20,21], a window of 4–6 min is recommended. This time to peak load should not include time required to perform unload-reload cycles.

If the test is being performed in CMOD control and data for Level II reduction are desired, at least one unload-reload cycle must be performed during the test, but before reaching peak load. Since peak load is not known accurately before the test is complete, several prepeak cycles will typically be performed. Unloading should continue to approximately 10% of the anticipated peak load. If the specimen unloads completely, the specimen and rollers will shift, which can cause a dangerous situation. Tests on concrete specimens sometimes become unstable during the postpeak portion of the test; therefore, when the applied load drops to approximately 97% of the peak load, conduct another unload-reload cycle. Waiting for the applied load to drop to 95% or 90% of the peak load might result in a missed opportunity for the post peak unload-reload cycle.

Data Reduction

In order to reduce the data from the RDB(B) specimens, the normalized stress intensity factor Y_{\min}^* must be determined. The details of determining Y_{\min}^* are presented in the second of this two-part paper [22]. For the RDB(B) with dimensions as specified in Fig. 4, Y_{\min}^* is 2.37. Most specimens made in the laboratory will have a slightly different W/B ratio. Assuming the notch geometry remains unchanged, the Y_{\min}^* value for W/B ratios from 1.98–2.05 are given by Eq 5. For specimens with a cut notch, the notch geometry might

differ from Fig. 4. The Y_{\min}^* value can be multiplied by a calibration factor C^* to account for these variations. The calibration factor depends upon the ratio of ligament length, $W - a_o$, to B . For ligament ratios of 1.45–1.55, the C^* value is given by Eq 6. The modified Y_{\min}^* value should be used in Eq 2 for Level I data reduction.

$$Y_{\min}^* = 0.740 \times (W/B) + 0.894 \quad (5)$$

$$C^* = 1.567 - 0.375 \times \left(\frac{W - a_o}{B} \right) \quad (6)$$

The inelastic correction factor p used for Level II data reduction is based on the load line displacement, LLD. For the compression loaded RDB, however, this measurement is extremely difficult to make accurately [19]. A much more accurate measurement is CMOD. Numerical simulations of RDB(B) specimens that explicitly represent the fracture process zone with a cohesive model have been performed to demonstrate that values of p calculated based on CMOD are reasonably close to values based on LLD. The simulations are explained in detail in Hanson [19] and are summarized in Table 3. The cohesive model dictates that the cohesive stress reduces to zero at a certain value of crack opening displacement $CMOD_{\max}$.

The resulting procedure for data reduction for RDB(B) specimens begins with calculating the apparent Level I fracture toughness using the appropriate Y_{\min}^* value and peak load in Eq 2. Then calculate the inelastic correction factor p from the load versus CMOD data using the two unload-reload cycles that most closely surround the peak load. Finally, calculate the Level II fracture toughness K_{Ic} using Eq 4.

Laboratory Results

In order to verify the performance of the compression loaded round double beam specimen, an extensive laboratory program was undertaken. The results for a variety of specimens sizes and concrete mixes are presented here.

Test Program

Three sizes of RDB specimens were used. The specimens had nominal diameters of 152, 305, and 610 mm. Where possible, all three sizes were made and tested for each batch of concrete. Be-

TABLE 3—Comparison of p values from load line displacement and crack mouth opening displacement for simulated RDB(B) tests.

Specimen Diameter (mm)	Cohesive Zone Properties			p factor		Effect on K_{Ic}^{measured}
	σ_t (MPa)	K_{Ic} (MPa \sqrt{m})	$CMOD_{\max}$ (mm)	LLD	CMOD	
152	2.17	1.10	0.269	0.28	0.29	1.1%
	2.17	4.40	0.645	0.37	0.34	–3.4%
	9.31	1.10	0.063	0.32	0.35	3.4%
	9.31	4.40	0.151	0.06	0.08	2.0%
305	2.17	1.10	0.269	0.36	0.37	1.2%
	2.17	4.40	0.645	0.29	0.29	0.0%
	9.31	1.10	0.063	0.10	0.13	3.1%
	9.31	4.40	0.151	0.05	0.07	2.0%
610	2.17	1.10	0.269	0.37	0.39	2.4%
	2.17	4.40	0.645	0.20	0.21	1.0%
	9.31	1.10	0.063	0.04	0.05	1.0%
	9.31	4.40	0.151	0.05	0.06	1.0%

TABLE 4—Summary of laboratory test results using the RDB(B) to measure K_{Ic} for concrete.

w/c Ratio	Max. Nom. Aggregate (mm)	Cast at (MPa)	f_t	RDB Diam. (mm)	Type of Notch	Age at Testing (days)	No. of Tests	K_{IQ} (Eq 2) (MPa \sqrt{m})	K_{Ic} (Eq 4) (MPa \sqrt{m})	COV for K_{Ic}
0.73	25	CWRU	3.60	152	cut	368	4	1.49	2.00	19.2%
0.60	10	CWRU	4.37	152	cut	374	4	1.50	1.86	3.2%
0.58	19	Cornell	3.25	152	cast	489	5	1.25	1.73	6.7%
				152	cut	490	5	1.46	1.79	4.6%
				305	cast	497	4	1.66	2.16	6.4%
				610	cast	498	5	1.56	2.02	13.8%
0.58	19	Cornell	3.21	152	cast	402	5	1.17	1.50	14.7%
				152	cut	402	5	1.36	1.84	9.6%
				305	cast	394	5	1.45	1.87	7.9%
				610	cast	393	5	1.31	1.59	8.5%
0.58	19	Cornell	3.10	305	cast	29	3	1.30	1.61	14.3%
			2.68	152	cast	118	2	1.07	1.57	2.2%
				610	cast	117	4	1.34	1.77	8.2%
			2.50	152	cast	502	2	1.31	1.75	4.0%
				152	cut	492	4	1.41	1.80	6.7%
0.56	10	Cardiff	1.99	152	cut	28	6	0.96	1.34	7.7%
0.40	10	Cardiff	3.85	152	cut	29	6	1.45	1.97	5.6%
0.35	10	CWRU	4.43	152	cut	331	4	1.36	1.71	4.1%
0.35	25	CWRU	3.74	152	cut	342	4	1.21	1.57	8.9%
0.32	13	Cornell	4.21	152	cut	32	3	1.27	1.62	0.7%
0.30	10	CWRU	4.91	152	cut	386	4	1.51	1.87	6.3%
0.30	25	CWRU	4.94	152	cut	380	4	1.46	1.96	13.2%
0.29	10	Cardiff	4.64	152	cut	28	4	1.58	1.93	6.2%
0.26	19	Cornell	4.20	152	cast	27	5	1.29	1.84	3.2%
				152	cut	27	5	1.37	1.90	17.7%
				305	cast	28	5	1.61	2.15	5.8%
				610	cast	29	5	1.83	2.33	7.3%
0.21	19	Cornell	4.35	152	cast	29	6	1.37	1.93	8.4%
				305	cast	28	3	1.62	2.10	4.1%

cause the mix design of concrete has a significant impact on the mechanical properties of the hardened material, several different concrete mixes were used. The concrete specimens were cast at Cornell University, University of Wales-Cardiff, and Case Western Reserve University, but all specimens were tested at Cornell. The mixes used a variety of nominal maximum aggregate sizes from 10–25 mm, and the water/cement ratios ranged from 0.21–0.70. All test specimens were sealed after casting to prevent loss or addition of moisture. Many specimens had a cast-in notch, but most of the 152 mm specimens had cut notches.

Test Results

The results of the laboratory test program are provided in detail in Hanson [19] or at <http://www.cfg.cornell.edu> as part of the International Collaboration for Fracture Toughness Testing of Concrete. All of the results are summarized here in Table 4. The summary includes averages of the K_{IQ} (Level I) and K_{Ic} (Level II) values as well as average splitting tensile strengths (ASTM C 496).

None of the RDB(B) specimens tested resulted in the crack deviating from the intended plane. Therefore, all of the test specimens produced usable data. In addition, the coefficients of variation of the results indicate good repeatability. Additionally, cutting the notch in the 152 mm diameter specimens does not appear to affect the K_{Ic} value.

Fracture toughness values from RDB(W) and RDB(B) tests are not available for a single batch of concrete. However, the RDB(W) results in Table 1 are comparable to the RDB(B) results in Table 4. Therefore, there is no indication of a systematic difference in fracture toughness values obtained from wedge loading and those from compression loading the RDB specimen.

Conclusions

The round double beam is an established test specimen for measuring the fracture properties of some materials. The common loading techniques of tension or wedging do not work well when testing materials with a relatively large characteristic length, such as concrete. In such cases, the crack tends to depart from the intended plane, which renders the data unusable. For these materials, the round double beam can be loaded in eccentric compression; the ASTM designation for this combination is RDB(B).

Test equipment, test procedures, and data reduction have been detailed for the RDB(B). Results from over 100 tests on concrete using the RDB(B) indicate that the combination of test specimen geometry and loading technique produces usable, repeatable data. The RDB(B) should also be useful for testing other materials with relatively large characteristic lengths.

More detailed information about this study including the raw data from the laboratory experiments and a draft test standard can be found at <http://www.cfg.cornell.edu>.

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