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Short Communication

Analysis of the asymmetric apodization using the fractional Fourier transform

S. GRANIERI†, D. ZALVIDEA and E. E. SICRE

Centro de Investigaciones Opticas (CIOp), PO Box 124, (1900) La Plata, Argentina

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Abstract. The effect of asymmetric apodization is analysed using the fractional Fourier transform. The focusing properties of an apodizing pupil mask are investigated by means of a simple display in a generalized phase-space (or x–p domain). Some comparative computer simulations are performed and curves displaying the Strehl ratio versus defocus are presented in order to illustrate the possibilities of our approach.

1. Introduction

Several efforts to improve the quality of the point spread function (PSF) of an optical imaging system have been done by using apodizer apertures. The purpose of the apodization process is to produce the suppression of the secondary maxima, or side lobes, of a diffraction pattern in order to enhance the resolving power of the optical system. It is a classical problem and it has been widely studied [1, 2]. In the apodization method proposed by Cheng and Siu [3], the aperture is modified in an asymmetric mode. This asymmetric aperture function produces the suppression of the secondary maxima at a side of the central peak of the diffraction pattern at the cost of increasing the lobes at the opposite side [4].

The fractional Fourier transform (FRT) was introduced in optical signal processing by Ozaktas and Mendlovic [5] and Lohmann [6]. Two different optical definitions of the FRT have been given. In the first, the FRT was defined based on light propagation in quadratic graded index media. In the second definition, the FRT of fractional order p results from a phase-space rotation of the input Wigner distribution function by an angle of $p\pi/2$. The FRT is a generalization of the classical Fourier transform and the information content stored in the FRT changes from spatial to spectral as the fractional order varies from $p = 0$ to $p = 1$. The relation between a FRT of order p and the free-space diffraction [7, 8] allows the analysis of the tolerance of optical imaging systems to focus errors and/or aberrations. There are several criteria to analyse the performance of an optical imaging system based on the on-axis irradiance. The Strehl ratio (SR), defined as the intensity values on axis at the diffraction focus conveniently normalized, is an important image quality parameter [9]. The purpose of this letter is to analyse the

† Also with the UID : OPTIMO, Facultad de Ingeniería Nacional de La Plata, La Plata, Argentina.

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effect of asymmetric apodizers on the focusing properties of an optical imaging system using the relationship between the fractional Fourier transform and the SR.

2. The FRT approach to Strehl ratio

By referring to figure 1, the optical imaging system denoted by L gives rise to a field amplitude around the image focal plane, located at $z = 0$, which can be expressed as

$$u(x; z) = \int_{-\infty}^{\infty} t_0(\xi) \exp\left(-\frac{i\pi}{\lambda f}\xi^2\right) \exp\left[\frac{i\pi}{\lambda(f+z)}(\xi - x)^2\right] d\xi, \tag{1}$$

where f is the image focal distance, and $t_0(\xi)$ is the one-dimensional version of the exit pupil function. The Wigner distribution function (WDF) associated with the field amplitude $u(x; z)$ is defined as

$$W_u(x, \nu; z) = \iint_{-\infty}^{\infty} u\left(x + \frac{x'}{2}; z\right) u^*\left(x - \frac{x'}{2}; z\right) \exp\left[-2\pi i x'\nu\right] dx'. \tag{2}$$

By using some of the well-known properties of the WDF [10], equation (1) can be rewritten as

$$W_u(x, \nu; z) = W_{t_0}\left(x - \lambda(f+z)\nu, \frac{x}{\lambda f} - \frac{z}{f}\nu\right). \tag{3}$$

Taking into account that the optical intensity can be found from the WDF spatial frequency projection, the on-axis intensity ($x = 0$) for varying z, or Strehl ratio versus defocus, is obtained as

$$S(z) = \frac{I(z)}{I(0)} = \frac{\int_{-\infty}^{\infty} W_{t_0}(-\lambda(f+z)\nu, -\nu z/f)\, d\nu}{\int_{-\infty}^{\infty} W_{t_0}(-\lambda f\nu, 0)\, d\nu}. \tag{4}$$

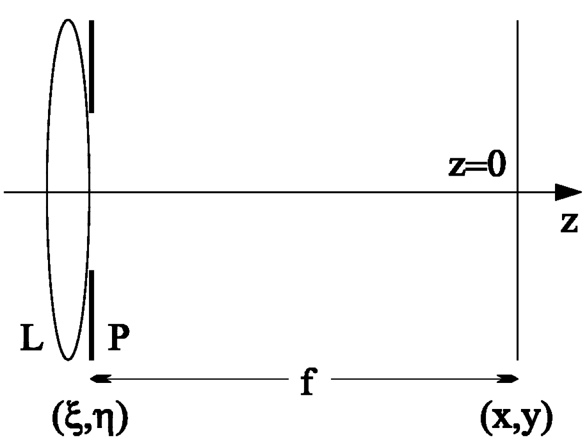

Figure 1. Optical imaging system: the several components are represented by an unique lens L; P is the exit pupil.

Based on the WDF, an integral operator for a FRT of order p can be obtained by analysing the propagation of an input signal $t_0(\xi)$ through the optical single-lens system of [6]. The FRT of $t_0(\xi)$ is obtained at the output plane by means of the Fresnel integral

$$u_p(x) = \int_{-\infty}^{\infty} t_0(\xi) \exp\left[\frac{\mathrm{i}\pi(x^2+\xi^2)}{\lambda f_1 \tan\phi}\right] \exp\left[\frac{-2\pi\mathrm{i}(x\xi)}{\lambda f_1 \sin\phi}\right] \mathrm{d}\xi, \tag{5}$$

f_1 being a free scale parameter and $\phi = p\pi/2$. The FRT modulus squared for varying x and p is known as (x–p) representation or Radon–Wigner transform of $t_0(\xi)$ [11]. For $x = 0$, it can be written as [10]:

$$\left|u_p(0)\right|^2 = \int_{-\infty}^{\infty} W_{t_0}(-\lambda f\nu \sin\phi, \nu\cos\phi)\,\mathrm{d}\nu. \tag{6}$$

By properly normalizing equation (6), the following relationship results

$$S(z) = \left|u_p(0)\right|^2, \tag{7}$$

whenever

$$p = \frac{2}{\pi}\arctan\left(\frac{f+z}{-z}\right). \tag{8}$$

Therefore, the SR for varying defocus is stored along the axis $x = 0$ in the (x–p) phase-space representation of the pupil function $t_0(\xi)$.

3. Display of the Strehl ratio

In the single display of the x–p chart, the vertical axis is the spatial one-dimensional light distribution $u_p(x)$ and the horizontal axis is the order p of the FRT. As a result, all the FRTs of the exit pupil function $t_0(\xi)$ are calculated and displayed in a single picture.

In order to analyse the changes in the tolerance to defocus produced by asymmetric apodization we compare a one-dimensional transparent but finite size exit pupil and its asymmetric apodizated version. To perform the asymmetric apodization of the rectangular aperture of width a, two narrow strips, each one of width b, with opposite phase factors are attached at both sides. The exit pupil becomes

$$t_0(\xi) = \begin{cases} -\mathrm{i}, & -\frac{a}{2} \leqslant \xi < -\frac{a}{2} + b, \\ 1, & -\frac{a}{2} + b \leqslant \xi \leqslant \frac{a}{2} - b, \\ \mathrm{i}, & \frac{a}{2} - b < \xi \leqslant \frac{a}{2}. \end{cases} \tag{9}$$

Figure 2 shows the computer simulation of the x–p chart squared modulus of $t(\xi)$, for the cases: (i) uniform aperture with $a = 10.5$, $b = 0$; and (ii) asymmetric apodizated aperture with $a = 10.5$, $b = 0.58$. The value for the parameter b in the last case was chosen in order to minimize one lateral side lobe in the asymmetrically apodized PSF, as is shown in [4]. The PSF of the optical system can be obtained from slices of the x–p chart along the vertical axis for $p = 1$. Accordingly with equation (7), the projection of the x–p chart along the p axis gives

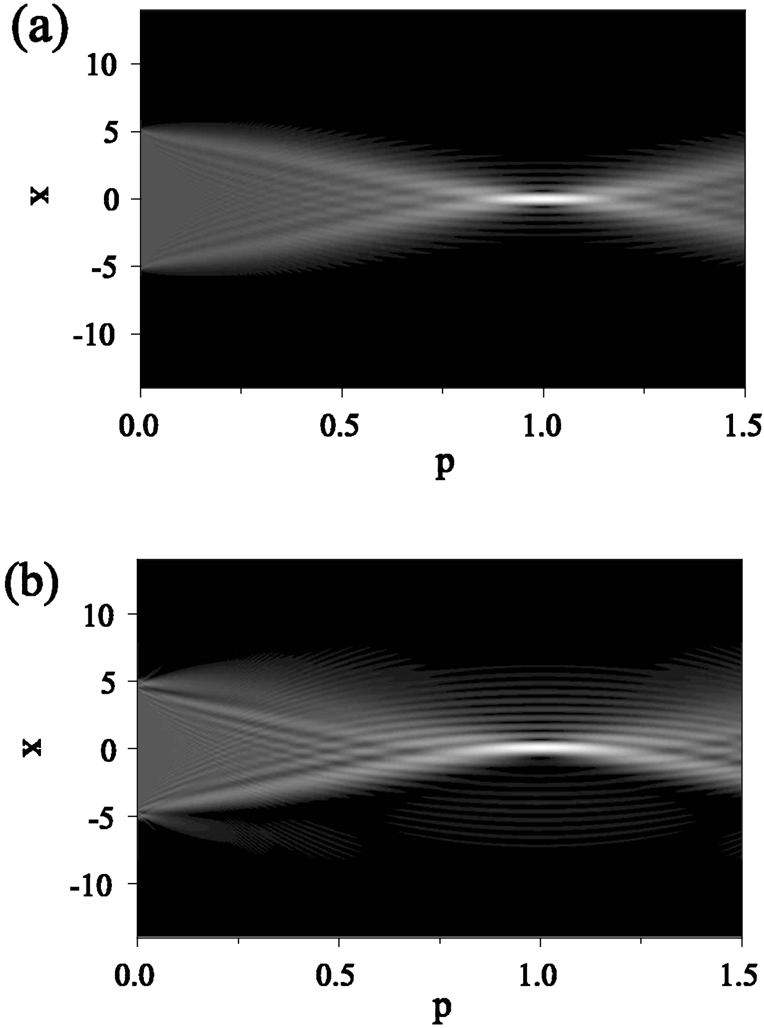

Figure 2. x–p chart squared modulus of the exit pupil $t_0(\xi)$: (*a*) unapodized version (case (i)); (*b*) asymmetrically apodized version (case (ii)).

the SR versus defocus distance z. Figure 3 (*a*) shows the PSF for the two studied cases. As can be expected, the lateral side lobe on the left side, for case (ii), is strongly suppressed. Finally, in figure 3 (*b*) the variation of the SR as a function of the defocus distance z is shown. In the case of the apodizing pupil mask, the maximum of the SR is decreased, but its tolerance to defocus is asymmetrically increased, with better performance for $z > 0$.

4. Conclusions

An approach based on the FRT was presented for analysing the tolerance to defocus of asymmetric apodizers. In a single two-dimensional display, known as the (x–p) chart, the PSF and the SR versus defocus are simultaneously stored along the perpendicular axis. Since this phase-space representation can be experi-

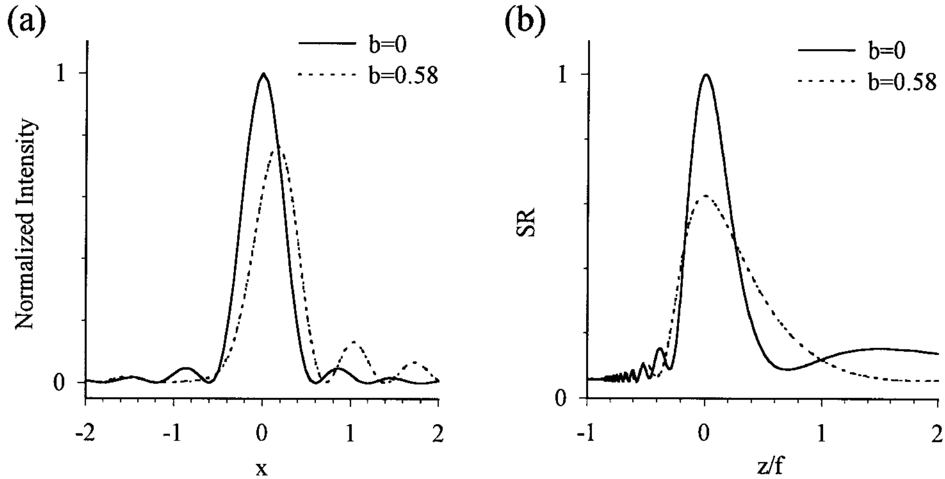

Figure 3. (*a*) The point spread functions (PSF) of $t_0(\xi)$ for the two analysed cases. (*b*) Strehl ratio (SR) versus defocus distance, z, for both considered cases.

mentally obtained [12], the method can be used to analyse the performance of different apodizing structures through their corresponding one-dimensional pupil versions.

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