

33.5 Polarization

Polarization is a characteristic of all transverse waves. This chapter is about light, but to introduce some basic polarization concepts, let's go back to the transverse waves on a string that we studied in Chapter 15. For a string that in equilibrium lies along the x -axis, the displacements may be along the y -direction, as in Fig. 33.21a. In this case the string always lies in the xy -plane. But the displacements might instead be along the z -axis, as in Fig. 33.21b; then the string always lies in the xz -plane.

When a wave has only y -displacements, we say that it is **linearly polarized** in the y -direction; a wave with only z -displacements is linearly polarized in the z -direction. For mechanical waves we can build a **polarizing filter**, or **polarizer**, that permits only waves with a certain polarization direction to pass. In Fig. 33.21c the string can slide vertically in the slot without friction, but no horizontal motion is possible. This filter passes waves that are polarized in the y -direction but blocks those that are polarized in the z -direction.

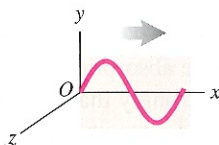
This same language can be applied to electromagnetic waves, which also have polarization. As we learned in Chapter 32, an electromagnetic wave is a *transverse* wave; the fluctuating electric and magnetic fields are perpendicular to each other and to the direction of propagation. We always define the direction of polarization of an electromagnetic wave to be the direction of the *electric*-field vector \vec{E} , not the magnetic field, because many common electromagnetic-wave detectors respond to the electric forces on electrons in materials, not the magnetic forces. Thus the electromagnetic wave described by Eq. (32.17),

$$\vec{E}(x, t) = \hat{j}E_{\max} \cos(kx - \omega t)$$

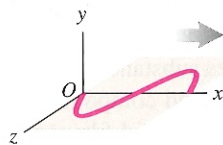
$$\vec{B}(x, t) = \hat{k}B_{\max} \cos(kx - \omega t)$$

33.21 (a), (b) Polarized waves on a string. (c) Making a polarized wave on a string from an unpolarized one using a polarizing filter.

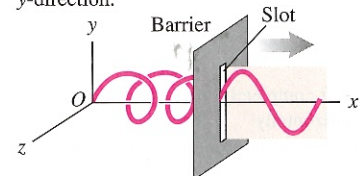
(a) Transverse wave linearly polarized in the y -direction



(b) Transverse wave linearly polarized in the z -direction



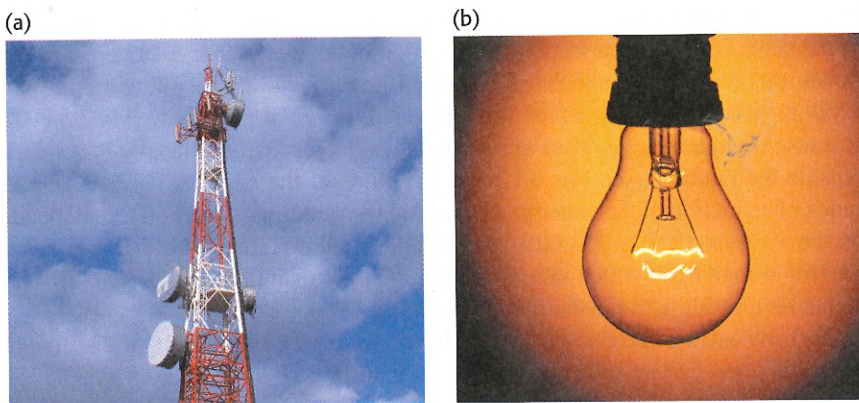
(c) The slot functions as a polarizing filter, passing only components polarized in the y -direction.



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33.22 (a) Electrons in the red and white broadcast antenna oscillate vertically, producing vertically polarized electromagnetic waves that propagate away from the antenna in the horizontal direction. (The small gray antennas are for relaying cellular phone signals.) (b) No matter how this light bulb is oriented, the random motion of electrons in the filament produces unpolarized light waves.



is said to be polarized in the y -direction because the electric field has only a y -component.

CAUTION The meaning of “polarization” It’s unfortunate that the same word “polarization” that is used to describe the direction of \vec{E} in an electromagnetic wave is also used to describe the shifting of electric charge within a body, such as in response to a nearby charged body; we described this latter kind of polarization in Section 21.2 (see Fig. 21.7). You should remember that while these two concepts have the same name, they do not describe the same phenomenon. \blacksquare

Polarizing Filters

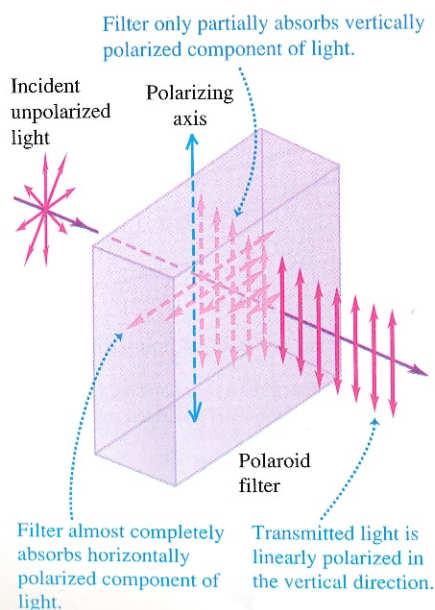
Waves emitted by a radio transmitter are usually linearly polarized. The vertical antennas that are used for radio broadcasting emit waves that, in a horizontal plane around the antenna, are polarized in the vertical direction (parallel to the antenna) (Fig. 33.22a).

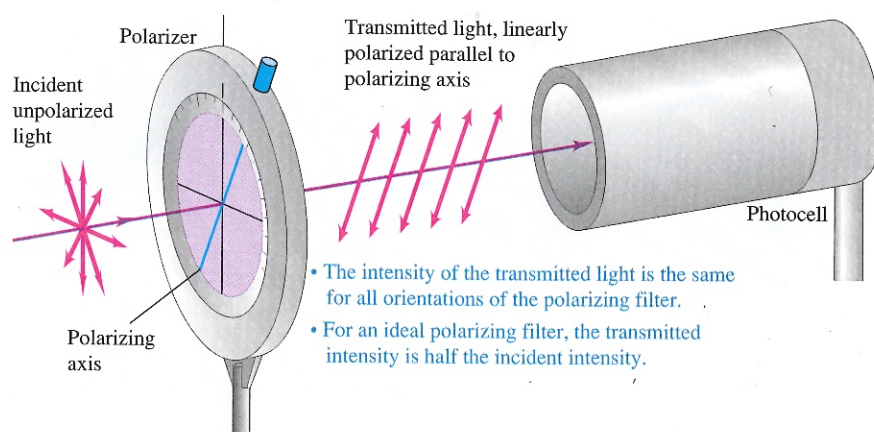
The situation is different for visible light. Light from incandescent light bulbs and fluorescent light fixtures is *not* polarized (Fig. 33.22b). The “antennas” that radiate light waves are the molecules that make up the sources. The waves emitted by any one molecule may be linearly polarized, like those from a radio antenna. But any actual light source contains a tremendous number of molecules with random orientations, so the emitted light is a random mixture of waves linearly polarized in all possible transverse directions. Such light is called **unpolarized light** or **natural light**. To create polarized light from unpolarized natural light requires a filter that is analogous to the slot for mechanical waves in Fig. 33.21c.

Polarizing filters for electromagnetic waves have different details of construction, depending on the wavelength. For microwaves with a wavelength of a few centimeters, a good polarizer is an array of closely spaced, parallel conducting wires that are insulated from each other. (Think of a barbecue grill with the outer metal ring replaced by an insulating one.) Electrons are free to move along the length of the conducting wires and will do so in response to a wave whose \vec{E} field is parallel to the wires. The resulting currents in the wires dissipate energy by I^2R heating; the dissipated energy comes from the wave, so whatever wave passes through the grid is greatly reduced in amplitude. Waves with \vec{E} oriented perpendicular to the wires pass through almost unaffected, since electrons cannot move through the air between the wires. Hence a wave that passes through such a filter will be predominantly polarized in the direction perpendicular to the wires.

The most common polarizing filter for visible light is a material known by the trade name Polaroid, widely used for sunglasses and polarizing filters for camera lenses. Developed originally by the American scientist Edwin H. Land, this material incorporates substances that have **dichroism**, a selective absorption in which one of the polarized components is absorbed much more strongly than the other (Fig. 33.23). A Polaroid filter transmits 80% or more of the intensity of a wave that is polarized parallel to a certain axis in the material, called the **polarizing**

33.23 A Polaroid filter is illuminated by unpolarized natural light (shown by \vec{E} vectors that point in all directions perpendicular to the direction of propagation). The transmitted light is linearly polarized along the polarizing axis (shown by \vec{E} vectors along the polarization direction only).





33.24 Unpolarized natural light is incident on the polarizing filter. The photocell measures the intensity of the transmitted linearly polarized light.

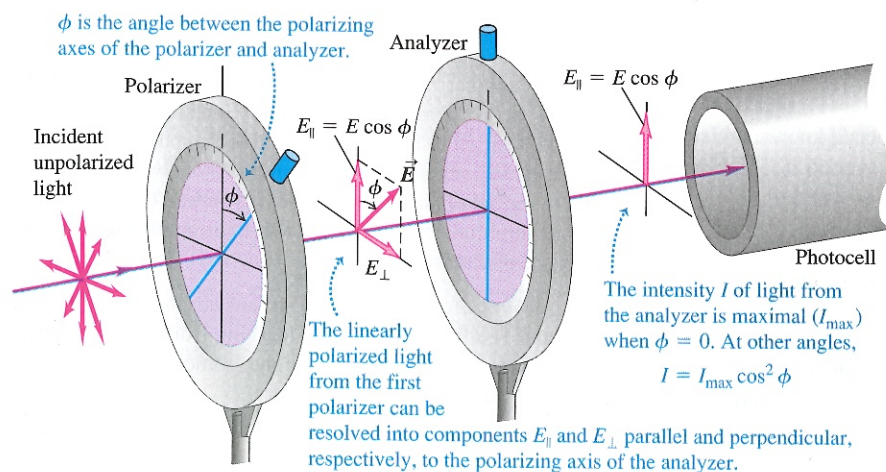
axis, but only 1% or less for waves that are polarized perpendicular to this axis. In one type of Polaroid filter, long-chain molecules within the filter are oriented with their axis perpendicular to the polarizing axis; these molecules preferentially absorb light that is polarized along their length, much like the conducting wires in a polarizing filter for microwaves.

Using Polarizing Filters

An *ideal* polarizing filter (polarizer) passes 100% of the incident light that is polarized parallel to the filter's polarizing axis but completely blocks all light that is polarized perpendicular to this axis. Such a device is an unattainable idealization, but the concept is useful in clarifying the basic ideas. In the following discussion we will assume that all polarizing filters are ideal. In Fig. 33.24 unpolarized light is incident on a flat polarizing filter. The \vec{E} vector of the incident wave can be represented in terms of components parallel and perpendicular to the polarizer axis (shown in blue); only the component of \vec{E} parallel to the polarizing axis is transmitted. Hence the light emerging from the polarizer is linearly polarized parallel to the polarizing axis.

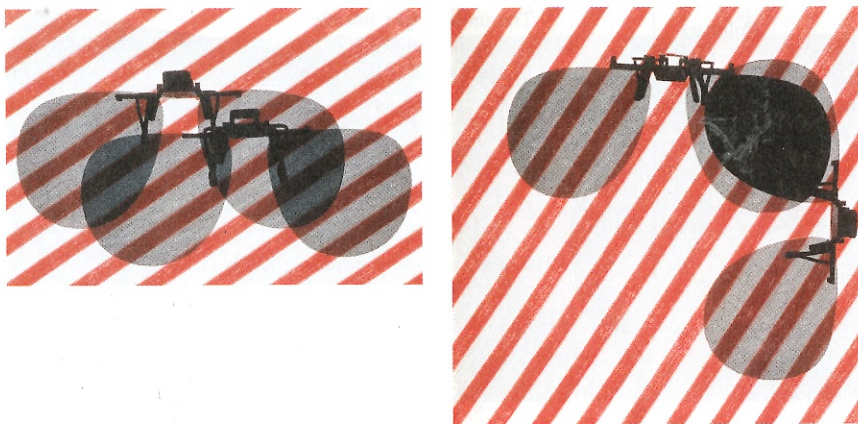
When unpolarized light is incident on an ideal polarizer as in Fig. 33.24, the intensity of the transmitted light is *exactly half* that of the incident unpolarized light, no matter how the polarizing axis is oriented. Here's why: We can resolve the \vec{E} field of the incident wave into a component parallel to the polarizing axis and a component perpendicular to it. Because the incident light is a random mixture of all states of polarization, these two components are, on average, equal. The ideal polarizer transmits only the component that is parallel to the polarizing axis, so half the incident intensity is transmitted.

What happens when the linearly polarized light emerging from a polarizer passes through a second polarizer, or *analyzer*, as in Fig. 33.25? Suppose the polarizing axis of the analyzer makes an angle ϕ with the polarizing axis of the



33.25 An ideal analyzer transmits only the electric field component parallel to its transmission direction (that is, its polarizing axis).

33.26 These photos show the view through Polaroid sunglasses whose polarizing axes are (left) aligned ($\phi = 0$) and (right) perpendicular ($\phi = 90^\circ$). The transmitted intensity is greatest when the axes are aligned; it is zero when the axes are perpendicular.



first polarizer. We can resolve the linearly polarized light that is transmitted by the first polarizer into two components, as shown in Fig. 33.25, one parallel and the other perpendicular to the axis of the analyzer. Only the parallel component, with amplitude $E \cos \phi$, is transmitted by the analyzer. The transmitted intensity is greatest when $\phi = 0$, and it is zero when the polarizer and analyzer are *crossed* so that $\phi = 90^\circ$ (Fig. 33.26). To determine the direction of polarization of the light transmitted by the first polarizer, rotate the analyzer until the photocell in Fig. 33.25 measures zero intensity; the polarization axis of the first polarizer is then perpendicular to that of the analyzer.

To find the transmitted intensity at intermediate values of the angle ϕ , we recall from our energy discussion in Section 32.4 that the intensity of an electromagnetic wave is proportional to the *square* of the amplitude of the wave [see Eq. (32.29)]. The ratio of transmitted to incident *amplitude* is $\cos \phi$, so the ratio of transmitted to incident *intensity* is $\cos^2 \phi$. Thus the intensity of the light transmitted through the analyzer is

$$I = I_{\max} \cos^2 \phi \quad (\text{Malus's law, polarized light passing through an analyzer}) \quad (33.7)$$

where I_{\max} is the maximum intensity of light transmitted (at $\phi = 0$) and I is the amount transmitted at angle ϕ . This relationship, discovered experimentally by Etienne Louis Malus in 1809, is called **Malus's law**. Malus's law applies *only* if the incident light passing through the analyzer is already linearly polarized.

Problem-Solving Strategy 33.2 Linear Polarization

IDENTIFY *the relevant concepts:* In all electromagnetic waves, including light waves, the direction of polarization is the direction of the \vec{E} field and is perpendicular to the propagation direction. Problems about polarizers are therefore about the components of \vec{E} parallel and perpendicular to the polarizing axis.

SET UP *the problem* using the following steps:

1. Start by drawing a large, neat diagram. Label all known angles, including the angles of all polarizing axes.
2. Identify the target variables.

EXECUTE *the solution* as follows:

1. Remember that a polarizer lets pass only electric-field components parallel to its polarizing axis.
2. If the incident light is linearly polarized and has amplitude E and intensity I_{\max} , the light that passes through an ideal polarizer has amplitude $E \cos \phi$ and intensity $I_{\max} \cos^2 \phi$, where ϕ is

the angle between the incident polarization direction and the filter's polarizing axis.

3. Unpolarized light is a random mixture of all possible polarization states, so on the average it has equal components in any two perpendicular directions. When passed through an ideal polarizer, unpolarized light becomes linearly polarized light with half the incident intensity. Partially linearly polarized light is a superposition of linearly polarized and unpolarized light.
4. The intensity (average power per unit area) of a wave is proportional to the *square* of its amplitude. If you find that two waves differ in amplitude by a certain factor, their intensities differ by the square of that factor.

EVALUATE *your answer:* Check your answer for any obvious errors. If your results say that light emerging from a polarizer has greater intensity than the incident light, something's wrong: A polarizer can't add energy to a light wave.



Example 33.5 Two polarizers in combination

In Fig. 33.25 the incident unpolarized light has intensity I_0 . Find the intensities transmitted by the first and second polarizers if the angle between the axes of the two filters is 30° .

SOLUTION

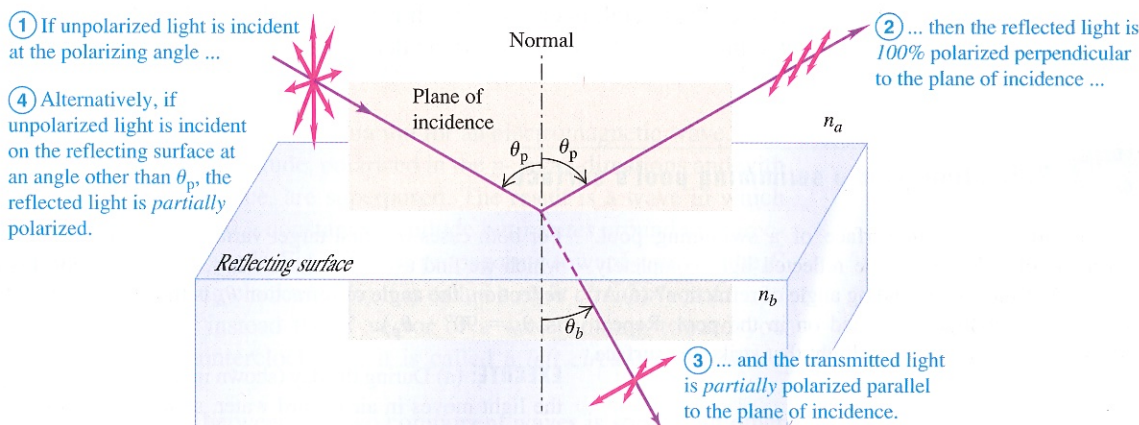
IDENTIFY and SET UP: This problem involves a polarizer (a polarizing filter on which unpolarized light shines, producing polarized light) and an analyzer (a second polarizing filter on which the polarized light shines). We are given the intensity I_0 of the incident light and the angle $\phi = 30^\circ$ between the axes of the polarizers. We use Malus's law, Eq. (33.7), to solve for the intensities of the light emerging from each polarizer.

EXECUTE: The incident light is unpolarized, so the intensity of the linearly polarized light transmitted by the first polarizer is $I_0/2$. From Eq. (33.7) with $\phi = 30^\circ$, the second polarizer reduces the intensity by a further factor of $\cos^2 30^\circ = \frac{3}{4}$. Thus the intensity transmitted by the second polarizer is

$$\left(\frac{I_0}{2}\right)\left(\frac{3}{4}\right) = \frac{3}{8}I_0$$

EVALUATE: Note that the intensity decreases after each passage through a polarizer. The only situation in which the transmitted intensity does *not* decrease is if the polarizer is ideal (so it absorbs none of the light that passes through it) and if the incident light is linearly polarized along the polarizing axis, so $\phi = 0$.

33.27 When light is incident on a reflecting surface at the polarizing angle, the reflected light is linearly polarized.



Polarization by Reflection

Unpolarized light can be polarized, either partially or totally, by *reflection*. In Fig. 33.27, unpolarized natural light is incident on a reflecting surface between two transparent optical materials. For most angles of incidence, waves for which the electric-field vector \vec{E} is perpendicular to the plane of incidence (that is, parallel to the reflecting surface) are reflected more strongly than those for which \vec{E} lies in this plane. In this case the reflected light is *partially polarized* in the direction perpendicular to the plane of incidence.

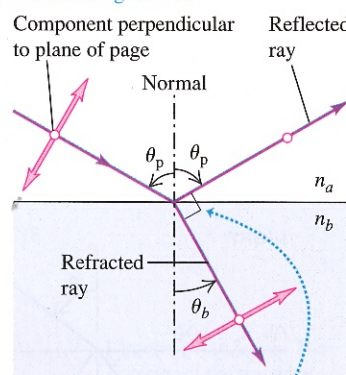
But at one particular angle of incidence, called the **polarizing angle** θ_p , the light for which \vec{E} lies in the plane of incidence is *not reflected at all* but is completely refracted. At this same angle of incidence the light for which \vec{E} is perpendicular to the plane of incidence is partially reflected and partially refracted. The *reflected* light is therefore *completely* polarized perpendicular to the plane of incidence, as shown in Fig. 33.27. The *refracted* (transmitted) light is *partially* polarized parallel to this plane; the refracted light is a mixture of the component parallel to the plane of incidence, all of which is refracted, and the remainder of the perpendicular component.

In 1812 the British scientist Sir David Brewster discovered that when the angle of incidence is equal to the polarizing angle θ_p , the reflected ray and the refracted ray are perpendicular to each other (Fig. 33.28). In this case the angle of refraction θ_b equals $90^\circ - \theta_p$. From the law of refraction,

$$n_a \sin \theta_p = n_b \sin \theta_b$$

33.28 The significance of the polarizing angle. The open circles represent a component of \vec{E} that is perpendicular to the plane of the figure (the plane of incidence) and parallel to the surface between the two materials.

Note: This is a side view of the situation shown in Fig. 33.27.



When light strikes a surface at the polarizing angle, the reflected and refracted rays are perpendicular to each other and

$$\tan \theta_p = \frac{n_b}{n_a}$$

so we find

$$n_a \sin \theta_p = n_b \sin(90^\circ - \theta_p) = n_b \cos \theta_p$$

$$\tan \theta_p = \frac{n_b}{n_a} \quad (\text{Brewster's law for the polarizing angle}) \quad (33.8)$$

This relationship is known as **Brewster's law**. Although discovered experimentally, it can also be *derived* from a wave model using Maxwell's equations.

Polarization by reflection is the reason polarizing filters are widely used in sunglasses (Fig. 33.26). When sunlight is reflected from a horizontal surface, the plane of incidence is vertical, and the reflected light contains a preponderance of light that is polarized in the horizontal direction. When the reflection occurs at a smooth asphalt road surface or the surface of a lake, it causes unwanted glare. Vision can be improved by eliminating this glare. The manufacturer makes the polarizing axis of the lens material vertical, so very little of the horizontally polarized light reflected from the road is transmitted to the eyes. The glasses also reduce the overall intensity of the transmitted light to somewhat less than 50% of the intensity of the unpolarized incident light.

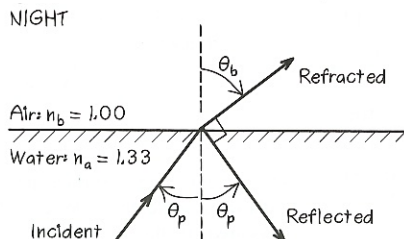
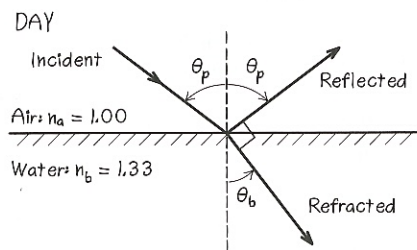
Example 33.6 Reflection from a swimming pool's surface

Sunlight reflects off the smooth surface of a swimming pool. (a) For what angle of reflection is the reflected light completely polarized? (b) What is the corresponding angle of refraction? (c) At night, an underwater floodlight is turned on in the pool. Repeat parts (a) and (b) for rays from the floodlight that strike the surface from below.

SOLUTION

IDENTIFY and SET UP: This problem involves polarization by reflection at an air–water interface in parts (a) and (b) and at a water–air interface in part (c). Figure 33.29 shows our sketches.

33.29 Our sketches for this problem.



For both cases our first target variable is the polarizing angle θ_p , which we find using Brewster's law, Eq. (33.8). For this angle of reflection, the angle of refraction θ_b is the complement of θ_p (that is, $\theta_b = 90^\circ - \theta_p$).

EXECUTE: (a) During the day (shown in the upper part of Fig. 33.29) the light moves in air toward water, so $n_a = 1.00$ (air) and $n_b = 1.33$ (water). From Eq. (33.8),

$$\theta_p = \arctan \frac{n_b}{n_a} = \arctan \frac{1.33}{1.00} = 53.1^\circ$$

(b) The incident light is at the polarizing angle, so the reflected and refracted rays are perpendicular; hence

$$\theta_b = 90^\circ - \theta_p = 90^\circ - 53.1^\circ = 36.9^\circ$$

(c) At night (shown in the lower part of Fig. 33.29) the light moves in water toward air, so now $n_a = 1.33$ and $n_b = 1.00$. Again using Eq. (33.8), we have

$$\theta_p = \arctan \frac{1.00}{1.33} = 36.9^\circ$$

$$\theta_b = 90^\circ - 36.9^\circ = 53.1^\circ$$

EVALUATE: We check our answer in part (b) by using Snell's law, $n_a \sin \theta_a = n_b \sin \theta_b$, to solve for θ_b :

$$\sin \theta_b = \frac{n_a \sin \theta_p}{n_b} = \frac{1.00 \sin 53.1^\circ}{1.33} = 0.600$$

$$\theta_b = \arcsin(0.600) = 36.9^\circ$$

Note that the two polarizing angles found in parts (a) and (c) add to 90° . This is *not* an accident; can you see why?