Asymptotic Analysis for Real-time Java Scoped-memory Areas

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Abstract

Java has recently joined C and C++ as a development platform for real-time and embedded applications. Java’s garbage collection, while generally a useful feature, can be problematic for these applications: garbage collection occurs at unpredictable times and its latency is typically unbounded. This can compromise necessary real-time guarantees.

To overcome these limitations, the Real-Time for Java Expert Group (RTJEG) proposed the Real-Time Specification for Java (RTSJ), which introduced new memory models and new threads to utilize those models. One such memory model uses scoped-memory areas, which work best in the context of a NoHeapRealtimeThread (NHRT). Although much work has been done with scoped-memory areas and NHRTs, there is no system-independent analysis of their costs. In this article we present an asymptotic analysis for RTSJ scoped-memory areas and NHRTs.

Keywords: Scoped Memory, Real-Time Java, Memory Management, Programming Languages, and Performance Analysis

1 INTRODUCTION

Real-time applications require bounded-time memory-management performance. Many real-time applications are written in languages that do not offer garbage-collection capabilities by default; as such, these applications do not suffer from the overhead associated with garbage collection. Since the advent of Java in 1995 [1, 6], programmers have been exploring ways to use Java for real-time programming. Java has become attractive because of its automated memory-management capabilities and the ease with which it can be used to construct programs.

Java has become ubiquitous in many programming environments but not in real-time and embedded environments. To advance the use of Java as a real-time programming language, the Real-Time for Java Expert Group (RTJEG) has issued the Real-Time Specification for Java [3] (RTSJ). The RTSJ provides extensions to Java in support of real-time programming [5] that do not change the basic structure of the language. Instead, they added new class libraries and Java Virtual Machine (JVM) extensions to the language. Compilers that are not optimized to take advantage of those extensions are not affected by their addition. Although the RTSJ revises many areas of the Java programming language, this article focuses on storage management.

In addition to the heap, where dynamic storage allocation occurs, the RTSJ specifies other memory areas for dynamic storage allocation. Those memory spaces are called immortal memory and scoped-memory areas [3]. Objects allocated in these memory areas are never subjected to garbage collection although the garbage collector may scan immortal memory. Objects in immortal memory are not collected until execution of the program completes. They are not collected earlier whether or not there are references to them. Objects in a scoped-memory area, on the other hand, are collected en masse when every thread in it exits that area. Scoped-memory areas are best used with NoHeapRealtimeThreads (NHRTs) [3].

Use of scoped-memory areas is not without additional cost or burden. Much work has been done with scoped-memory areas and NHRTs; however, to the best of our knowledge, there is no record in the literature of system-independent cost analysis for scoped-memory regions with NHRTs. The goal of this article is to provide a framework for comparing programming with RTSJ scoped memory to programming with other memory models, e.g., the heap. While there are several approaches that can be used to do this comparison, we propose a model that employs asymptotic analysis.

The rest of the paper is organized as follows. Section 2 discusses RTSJ’s influence on memory management. Sections 3, 4 and 5 provide analysis of scoped-memory areas. Section 6 offers concluding remarks and Section 7 acknowledges those who have assisted...
in preparation of the final draft of the paper.

2 RTSJ'S MEMORY MANAGEMENT

One of the most interesting features of the RTSJ is the new memory management model based on scoped-memory areas (or scopes for short) [7]. This model ensures programmers of timely reclamation of memory and predictable performance. This comes at the cost of learning an unfamiliar programming model—a restrictive model that relies on the use of scopes. These new scopes were designed to meet two very important requirements [7]: providing predictable allocation and deallocation performance, and ensuring that real-time threads do not block when memory is reclaimed by the virtual machine.

Figure 1: Scoped-memory single-parent rule. A is the parent of both B and C.

To meet these requirements, the RTSJ ensures that objects in a scope are not deallocated individually. Instead, the entire scope is collected en masse when all thread that have entered the scope have exited the scope. A scope is a pool of memory from which objects are allocated. Each scope can be entered by multiple threads. These threads can allocate objects in the memory pool and communicate with each other by shared variables. A new scope can also be instantiated by a thread executing within its current scope. This is known as the nesting of scopes. Such nesting, however, is controlled by the order of threads entering the scopes—see Figure 1. A scope can become the parent of multiple scopes but no scope is allowed to have multiple parents.

To take advantage of scopes, the RTSJ defined a new type of thread called NoHeapRealtimeThread (NHRT). NHRTs cannot allocate objects in the garbage-collected heap and they cannot reference objects in the heap. These constraints were added to prevent NHRTs from experiencing unbounded delay due to the locking of heap objects during garbage collection [5]. NHRTs have the highest priority among all threads and can preempt even the garbage collector.

Table 1 details how objects in certain memory areas are not allowed to reference objects in other memory areas. This constraint does not apply only to objects, but also to threads so that real-time threads do not block when the JVM reclaims objects.

3 SCOPED-MEMORY ANALYSIS

This section focuses on computing asymptotic bounds for the RTSJ scoped-memory model when NHRTs are used. These bounds will give an idea of how expensive it is to execute applications in a scoped-memory environment. They will also facilitate comparison of execution in scoped-memory environments with execution in other memory environments (e.g., the heap).

Asymptotic analysis of scoped memory

We present a model for computing the asymptotic bounds that follows the steps listed below.

1. Select an abstract data type (ADT) that can hold an arbitrary number of elements.
2. Define the fundamental operations of the ADT and provide an interface.
3. Propose at least one implementation for each operation.
4. Adopt methods from Cormen et al. [4] to compute the worst-case running time for each operation.
5. Perform step 4 for an intermixed sequence of n operations. This denotes the ADT problem (e.g., the stack problem).
6. Repeat step 1.

The input size n for each operation is characterized by the number of elements in the collection immediately before the operation is run. Should there be a need to use a different characterization for the input size of an operation, one will be provided. The pseudocode provided for selected operations follows the pseudocode conventions from Cormen et al. [4]. We apply this model to solve the problem at hand.

4 STACK ANALYSIS

The stack is the first abstract data type we analyze. A stack is an ADT that operates on the Last-In-First-Out (LIFO) principle. One end of a stack, called the top-of-stack, is used for each stack operation. The fundamental operations are:
<table>
<thead>
<tr>
<th>Objects in</th>
<th>Reference to Heap</th>
<th>Reference to Immortal</th>
<th>Reference to Scoped</th>
</tr>
</thead>
<tbody>
<tr>
<td>Heap</td>
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<td>Allowed</td>
<td>Not allowed</td>
</tr>
<tr>
<td>Immortal</td>
<td>Allowed</td>
<td>Allowed</td>
<td>Not allowed</td>
</tr>
<tr>
<td>Scoped</td>
<td>Allowed</td>
<td>Allowed</td>
<td>Allowed if same, outer, or shared scope</td>
</tr>
</tbody>
</table>

Table 1: References between storage areas [3] courtesy of Deters and Cytron [5]. Objects in the heap are allowed to reference objects in immortal memory, but not objects in scoped memory.

1. **IS-EMPTY(S)** - an operation that returns the binary value **TRUE** if stack S is empty, **FALSE** otherwise.

2. **PUSH(S, x)** - an operation that puts element x at the top of stack S.

3. **POP(S)** - an operation that removes the element at the top of the stack S and returns it. If the stack is empty the special pointer value **NULL** is returned. **NULL** is used to signify that a pointer has no target.

**Heap implementation**

Several data structures, including the singly linked list, can be used to implement a stack in the heap. The IS-EMPTY operation checks whether the top-of-stack points to NULL. The PUSH operation adds a new element to the top-of-stack, and the POP operation updates the top-of-stack and returns the topmost element. Each of these fundamental operations requires \( T(n) = O(1) \) time using a singly linked list.

**Scoped memory implementation**

For a scoped-memory implementation of stack we make the following assumptions:

1. Each application \( A \) that manages a stack \( S \) is fully compliant with the RTSJ.

2. \( A \) has a single thread \( T_a \), which is an instance of RTSJ NHRT.

3. \( A \) executes on an RTSJ-compliant JVM.

4. \( T_a \) can legally access \( S \) and the elements managed by \( S \).

5. Before an element \( x \) is pushed on stack \( S \), a new scope \( s \) must first be instantiated to store \( x \), and \( T_a \) must enter \( s \).

Assumption 5 is relevant for the purpose of complexity analysis. Although we do not suggest one scope per element for an actual implementation, here we are concerned about worst-case analysis. Storing a single element in a scope simplifies analysis and yields the smallest amount of unnecessarily live storage in scoped memories for this particular data structure.

Pseudocode and analysis for the fundamental stack operations follow.

**IS-EMPTY:** We assume that there is a **TOS** field in the current scope that points to the top-of-stack element. If the **TOS** field points to the stack object \( S \), then the application thread \( T_a \) is executing in the scope containing \( S \). Thus, \( S \) contains no elements, so the stack is empty. If \( c_1 \) is the time required to execute line 1 of IS-EMPTY then the worst-case running time of IS-EMPTY is \( T(n) = c_1 = O(1) \).

```
IS-EMPTY(S)
1 return TOS = S
```

Figure 2: Procedure to test if the stack is empty—scoped memory implementation

**PUSH:** The PUSH operation depicted in Figure 3 is equivalent to the following sequence of basic operations performed by the application thread \( T_a \). From the current scope \( T_a \) instantiates a new scope \( sm \). \( T_a \) enters \( sm \) then sets the **TOS** field in \( sm \) to point to element \( x \).

```
PUSH(S, x)
1  scope ← new ScopedMemory(m)
2  enter(scope, T_a)
3  TOS ← x
```

Figure 3: Procedure to push an element onto the stack—scoped memory implementation. \( m \geq |x| + |TOS| \).

Assuming each line \( i \) in PUSH requires \( c_i \) time for execution, the worst case execution time for PUSH is given by \( T(n) = c_1 + c_2 + c_3 = O(1) \). The correctness of this result is based on the fact that each line is executed once per invocation. Because a scope has a limited lifetime dictated by the reference count of threads executing therein, \( T_a \) is not allowed to exit \( sm \). To ensure that \( T_a \) keeps \( sm \) alive \( T_a \) does not return from the `enter()` method of line 2 of Figure 3. Should \( T_a \) return from the `enter()` method, the thread reference-count of \( sm \) would drop to zero, \( sm \) would be collected, and the PUSH operation would fail.


**POP**: The POP operation returns the TOS element if one exists, NULL otherwise. Assuming each line \( i \) of the POP operation (Figure 4) requires \( c_i \) time to execute, the worst-case execution time for the POP operation is given as \( T(n) = O(1) \).

```plaintext
POP(S)
1  if IS-EMPTY(S)
2    then x ← NULL
3  else x ← TOS
4  return x
```

Figure 4: Procedure to pop the topmost element off the stack—scoped memory implementation

After popping the stack, \( T_a \) must return from the `enter()` method of line 2 of Figure 3. We assume for all practical purposes that returning from the `enter()` method takes \( O(1) \) time. The new top-of-stack becomes the TOS element of the parent scope, i.e., the parent of the scope that contained the popped element.

**The stack problem**

The stack problem is defined as the problem of running an intermixed sequence of \( n \) PUSH and POP operations on a stack instance. We analyze the stack problem for a singly-linked-list implementation in the heap and for a scoped-memory implementation of stack. Let \( n \) denote the total number of operations and let \( m \) denote the number of PUSH operations. The number of POP operations is thus given by \( n - m \) where \( n - m \leq m \leq n \). The worst-case running time for the singly linked list implementation of the stack problem is computed as

\[
T(n) = T_{push}(m) + T_{pop}(n - m) \\
= m \times c_1 + (n - m) \times c_2 \\
= mc_1 + mc_2 - mc_2 \\
= (c_1 - c_2)m + c_2n \\
= O(n)
\]

For a scoped-memory implementation the running time for PUSH or POP is \( O(1) \). Thus, the running time for the stack problem in the context of a scoped memory implementation is given by \( T(n) = O(n) \).

**Discussion**

The linked-list implementation in the heap yields \( T(n) = O(1) \) worst-case execution time for each stack operation. The scoped memory implementation also yields \( T(n) = O(1) \) worst-case execution time for each operation. The stack problem, i.e., the problem of running an intermixed sequence of \( n \) PUSH and POP operations yields a worst-case running time of \( T(n) = O(n) \) for each implementation, as expected. Given a particular program that uses a stack, the programmer can thus choose among these two implementations.

Although a singly-linked-list implementation works well in the heap, pointer manipulation can affect the proportionality constants of the running time for each operation. Garbage collection can also interfere with the running times of stack operations if the application executes in a heap that is subject to garbage collection.

A scoped-memory implementation, while good in real-time environments, comes at the cost of learning a new, restrictive programming model. Real-time programmers, however, can benefit from the timing guarantees promised by the RTSJ.

**5 QUEUE ANALYSIS**

A queue is an ADT that operates on the First-In-First-Out (FIFO) principle. The fundamental operations for a queue are:

1. **ISQ-EMPTY(Q)** - an operation that returns the binary value `TRUE` if queue \( Q \) is empty, `FALSE` otherwise.
2. **ENQUEUE(Q, x)** - an operation that adds element \( x \) to the rear of queue \( Q \).
3. **DEQUEUE(Q)** - an operation that removes the element at the front of queue \( Q \) and returns it. If the queue is empty, NULL is returned.

**Heap implementation**

One possible implementation of the queue ADT in the heap uses a singly linked list data structure with two special pointers, `front` and `rear`. The ISQ-EMPTY operation checks whether `front` points to NULL. The ENQUEUE operation adds a new element to the rear end of the linked list and updates the `rear` pointer. The DEQUEUE operation updates the `front` pointer and returns the element that was at the front of the linked list. Each of these fundamental operations takes time \( T(n) = O(1) \).

**Scoped-memory implementation**

Consider execution of an application \( A \) that manages a queue instance in an RTSJ scoped-memory environment. Efficient execution of \( A \) depends on proper management of memory, which is a limited resource. Assume \( A \) uses a stack of scoped-memory instances to manage the queue. Assume also, for the purpose of worst-case analysis, that a queue element resides in its own scope when added to the queue. A service stack with its own NHRT \( T_0 \) is used to facilitate the ENQUEUE operation. See Figure 5 for a representation of a queue instance. If \( T_0 \) is the application
thread, then $T_0$ is a NHRT. Detailed analysis of the fundamental queue operations follows.

**DEQUEUE:** The DEQUEUE operation removes the element at the front of the queue and returns it if one exists. Otherwise, it returns NULL. A close examination of the DEQUEUE operation (Figure 7) reveals that it is similar to the POP operation of Figure 4. Hence, the worst-case running time for DEQUEUE is $T(n) = O(1)$ time.

![Figure 5: Queue representation in an RTSJ scoped-memory environment. Rounded rectangles represent scoped-memory instances and circles represent object instances. $T_0$ is the application thread and $T_1$ services the stack. The arrows pointing downward represent legal scope references. The sync field/object is a synchronization point for $T_0$ and $T_1$. $E_i$ denotes element $i$. On the queue, $E_i$ is a reference to element $i$.](image)

**ISQ-EMPTY:** The current scope contains a front field that points to the front of the queue. An empty queue is a queue with no elements. Emptiness, in Figure 6, is illustrated by the front field of the current scope pointing to the queue object itself. Assuming that the running time of the lone line of ISQ-EMPTY is $c_1$, the worst-case running time of ISQ-EMPTY is given as $T(n) = c_1 = O(1)$.

**ENQUEUE:** The ENQUEUE operation is a relatively complex operation because of the referencing constraints imposed by RTSJ: objects in an ancestor scope cannot reference objects in a descendant scope because the descendant scope is reclaimed before the ancestor scope. As a consequence of these constraints, the elements already in a queue must first be stored somewhere before a new element can be enqueued. After the element is enqueued, all the stored elements are put back on the queue in their correct order. A stack is an ideal structure to store the queue elements because it preserves their order for the queue. As illustrated in Figure 5, two threads are needed to facilitate the ENQUEUE operation: one for the queue and one to service the stack. The thread that services the queue is the application thread and is referred to as $T_0$; $T_1$ is the service thread for the stack. These two threads are synchronized by a parameter sync, which they use to share data between them—see Figure 8.

![Figure 6: Procedure to test if the queue is empty—scoped memory implementation](image)

**ENQUEUE(Q, x)**

1. while !ISQ-EMPTY(Q)  
2. do sync ← DEQUEUE(Q)  
3. PUSH(S, sync)  
4. $S_c ←$ new ScopedMemory($m$)  
5. enter($S_c$, $T_0$)  
6. front ← x  
7. while !IS-EMPTY(S)  
8. do sync ← POP(S)  
9. PUSH-Q(Q, sync)

**PUSH-Q(S, x)**

1. scope ← new ScopedMemory($m$)  
2. enter(scope, $T_0$)  
3. front ← x

**Figure 7: Procedure to remove an element from the front of the queue—scoped memory implementation**

**Figure 8: Procedure to add an element to the rear of the queue—scoped memory implementation**

**Figure 9: Private helper method that puts an element at the front of the queue in the same manner that an element is pushed onto a stack—scoped memory implementation. $m \geq |x| + |front|$**

**PUSH-Q is a private method that puts a stored element back on the queue in the way that the PUSH operation works for a stack. The worst-case running time for this method is $T(n) = O(1)$ time.**
same running time for the PUSH operation of Figure 3.

Given the procedure in Figure 8 the worst-case running time for ENQUEUE is linear in the number of elements already in the queue:

\[ T(n) = (n + 1)c_1 + nc_2 + nc_3 + c_4 + c_5 + c_6 + c_7 + nc_8 + nc_9 \]
\[ = (c_1 + c_2 + c_3 + c_7 + c_8 + c_9)n + c_1 + c_4 + c_5 + c_6 + c_7 \]
\[ = c_b * n + c_a \]
\[ = O(n) \]

The queue problem

The queue problem is defined as the problem of running an intermixed sequence of \( n \) ENQUEUE and DE-QUEUE operations on a queue instance. We compute the worst-case running time for each implementation. Suppose \( n \) denotes the number of operations in the sequence and \( m \) denotes the number of ENQUEUE operations, then the number of DEQUEUE operations is given as \( n - m \) where \( n - m \leq m \leq n \). The worst-case running time for the heap implementation is thus given as:

\[ T(n) = T_{enq}(m) + T_{deq}(n - m) \]
\[ = m * c_1 + (n - m) * c_2 \]
\[ = nc_2 + m(c_1 - c_2) \]
\[ = O(n) \]

This is identical to the linked-list analysis of the stack problem because the insertion operation an the deletion operation each executes in constant time. The scoped memory implementation is more complex for the ENQUEUE operation. The running time for the queue problem in that context is also more complex and more costly. We compute the worst-case running time as follows:

\[ T(n) = T_{enq}(m, \bar{s}) + T_{deq}(n - m) \]
\[ = T_{enq}(m, \bar{s}) + (n - m) * c_2 \]

\( \bar{s} = (s_1, s_2, \ldots, s_m) \) is included as input to the computation of the running time for the ENQUEUE operations because the running time of each invocation of the ENQUEUE operation depends on the number of elements in the queue; \( s_i \) denotes the number of elements on the queue before the \( i \)th operation. Given fixed \( n \) and \( m \), the worst-case running time for the sequence of operations occurs when no DEQUEUE operations precede an ENQUEUE operation. In this case the values in \( \bar{s} \) are monotonically increasing from 0 to \( m - 1 \); so for the computation of \( T(n) \) given below, \( s_i = i - 1 \). \( c_a \) and \( c_b \) are derived from the ENQUEUE analysis above and \( c_d \) is the time for the constant DE-QUEUE operation.

\[ T(n) = T_{enq}(m, \bar{s}) + T_{deq}(n - m) \]
\[ = \sum_{i=1}^{m} (c_a + s_i c_b) + T_{deq}(n - m) \]
\[ = \sum_{i=1}^{m} (c_a + (i - 1)c_b) + T_{deq}(n - m) \]
\[ = mc_a + \left( \sum_{i=1}^{m} c_b i \right) - mc_b + T_{deq}(n - m) \]
\[ = mc_a - mc_b + c_b \left( \sum_{i=1}^{m} i \right) + T_{deq}(n - m) \]
\[ = m(c_a - c_b) + c_b \frac{m(m + 1)}{2} + T_{deq}(n - m) \]
\[ = m(c_a - c_b) + \frac{m^2 c_b}{2} + \frac{mc_b}{2} + T_{deq}(n - m) \]
\[ = \frac{c_b}{2} m^2 + \frac{2c_a - c_b}{2} m + (n - m)c_d \]
\[ = \frac{c_b}{2} m^2 + \frac{2c_a - c_b - 2c_d}{2} m + c_d n \]

Since \( m \leq n \) it follows that \( T(n) = O(n^2) \). Thus, for a scoped-memory queue implementation the worst-case running time for an intermixed sequence of \( n \) ENQUEUE and DEQUEUE operations is \( T(n) = O(n^2) \).

Discussion

Two possible implementations for the queue ADT were suggested: a singly-linked-list implementation and an RTSJ scoped-memory implementation. The singly-linked-list implementation yields \( T(n) = O(1) \) worst-case execution time for each queue operation. The scoped-memory implementation yields \( T(n) = O(1) \) worst-case execution time for the ISQ-EMPTY and DEQUEUE operations, but \( O(n) \) time for the ENQUEUE operation. The reason the worst-case execution time for ENQUEUE is linear instead of constant is based on the referencing constraints imposed by RTSJ’s scoping rules. Scopes are usually instantiated in a stack-like fashion. Thus, to enqueue an element the scope stack must be popped and the element in each scope must be stored on a stack or some other data structure. A new scope to enqueue the element must then be instantiated from the base of the scope stack and be placed on the queue. The elements stored away for the ENQUEUE operation must then be restored on the queue in a LIFO manner.

In addition to performing analysis for each operation, we performed analysis for the queue problem, i.e., the problem of running a sequence of \( n \) ENQUEUE and DEQUEUE operations on a queue in-
The singly-linked-list implementation gives a worst-case running time of $O(n)$ and the scoped-memory implementation gives a worst-case running time of $O(n^2)$. Thus, the scoped-memory implementation gives a running time that is an order of magnitude larger than the running time given by the singly-linked-list implementation. This is rather expensive for an environment that governs its own memory and gives NHRTs higher priorities than any garbage collector.

**Improved scoped-memory implementation**

We presented thus far an implementation of a queue in an RTSJ scoped-memory environment that turned out to have a worst-case running time of $O(n^2)$ for $n$ consecutive ENQUEUE operations. Here, we present a modified queue implementation that has a worst-case running time of $O(n)$ for $n$ consecutive ENQUEUE operations. Here, we present a modified queue implementation that has a worst-case running time of $O(n)$ for $n$ consecutive ENQUEUE operations.

First, the elements already in the queue are copied over to the service stack. Then, not one but 16 nested scopes, each capable of holding one queue element, are created. The element being enqueued is placed in the most deeply nested scope, i.e., the one closest to the front of the queue. Then the 15 elements on the service stack are copied back over to the queue in their correct order. Now the next 15 ENQUEUE operations will fill the empty scopes without having to use the service stack. A field $n_{enq}$ in the synchronized shared memory (in the scope containing both the queue and service stack) will keep track of the number of times ENQUEUE has been called.

<table>
<thead>
<tr>
<th>line</th>
<th>time cost</th>
<th>freq. when $n_{enq} = 2^x$</th>
<th>freq. otherwise</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$c_1$</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>$c_2$</td>
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<td>6</td>
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</tbody>
</table>

Figure 11: Statistics for procedure in Figure 10. $n_{enq}$ is incremented as the first step of the ENQUEUE algorithm. Then it can be seen from Figure 11 that the 10th ENQUEUE call takes $c_a + c_b i$ time if $i = 2^x$ for some integer $x$, where

- $c_a = c_1 + c_2 - c_3 - c_5 + c_6 + c_{10} + c_{11} - c_{13} - c_{14}$
- $c_b = c_3 + c_4 + c_5 + c_6 + c_7 + c_8 + c_9 + c_{12} + c_{13} + c_{14}$

The queue now has 15 elements, each in its own scope. Then another ENQUEUE operation is to be performed. First, the elements already in the queue are copied over to the service stack. Then, not one but 16 nested scopes, each capable of holding one queue element, are created. The element being enqueued is placed in the most deeply nested scope, i.e., the one closest to the front of the queue. Then the 15 elements on the service stack are copied back over to the queue in their correct order. Now the next 15 ENQUEUE operations will fill the empty scopes without having to use the service stack. A field $n_{enq}$ in the synchronized shared memory (in the scope containing both the queue and service stack) will keep track of the number of times ENQUEUE has been called.

**The queue problem revisited:**

As with the previous implementation, we use a service stack with its own NHRT $T_1$ to manage the queue. We also limit each scope to holding at most one queue element, and the ISQ-EMPTY and DEQUEUE operations remain the same as those presented above. Whereas before we copied the entire queue over to the service stack for each ENQUEUE operation, now we do so only for the $i$th ENQUEUE operation when $i$ is a power of 2. After the queue elements are copied to the service stack, and before they are copied back to the queue in their previous order, we create not one but $i$ new scopes at the rear of the queue. The new element is enqueued in the deepest scope—the one nearest to the front of the queue. The other scopes remain empty until they are filled on subsequent ENQUEUE operations.

Say, for example, we start with an empty queue and perform 15 consecutive ENQUEUE operations.
and \(c_e\) time otherwise, where
\[
c_e = c_1 + c_2 + c_{15} + c_{16} + c_{17}
\]
So, assuming \(n = 2^x\) for some integer \(x\) (which is a worst case, since the last ENQUEUE will be a linear-time and not a constant-time operation), the total running time for all \(n\) ENQUEUES will be
\[
T(n) = \sum_{j=0}^{x} (c_a + c_b 2^x) + (2^x - x - 1)c_e
\]
\[
= (x + 1)c_a + \left( \sum_{j=0}^{x} 2^x \right) c_b + (2^x - x - 1)c_e
\]
\[
= (x + 1)c_a + (2^{x+1} - 1)c_b + (2^x - x - 1)c_e
\]
\[
= (2c_b + c_e)2^x + (c_a - c_e)x + c_a - c_b - c_e
\]
\[
= (2c_b + c_e)n + (c_a - c_e) \log_2 n + c_a - c_b - c_e
\]
\[
= O(n)
\]
Therefore the improved ENQUEUE operation has a worst-case running time of \(O(n)\) on the queue problem. However, because it overallocates when resizing, it relies at some point on having twice the number of cells allocated as are actually in use. Interestingly, a time/space tradeoff of this nature is endemic to the real-time collectors [2] as well.

6 CONCLUSIONS

There are many implementations of RTSJ including TymeSis, JRate, and Sun Microsystems’ implementation. Many in the real-time and academic communities are experimenting with RTSJ and scoped-memory areas. However, until now, there has not been an objective analysis of scoped-memory areas.

To fill this void we presented a model to perform asymptotic analysis for RTSJ scoped-memory areas. Using the model, we determined asymptotic bounds for RTSJ scoped-memory areas in the context of stacks and queues. Our analysis permits us to compare scoped memory with other memory models and to reason more thoroughly about different memory models.

One of the assumptions of our model is to consider one element per scope. Though we do not suggest that this assumption be followed in practice, the analysis does not change if multiple elements are allowed per scope. Consider, for example, \(4k\) elements per scope. If \(4k\) elements are enqueued on a queue and a \((4k + 1)\)th element is to be enqueued (with no intervening dequesues), a new scope would have to be instantiated to accommodate that element. That enqueue operation suffers the cost discussed in Section 5.

In the near future, we plan to explore RTSJ scoped memory use with other abstract data types to gain a more complete understanding of how they impact the cost of scoped memory use. However, even in the case of a simple queue, using RTSJ scoped memories instead of a collector does not avoid the time/space tradeoffs experienced with the real-time collectors. Ours is the first work to point this out.

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References