

Name \_\_\_\_\_ Section \_\_\_\_\_

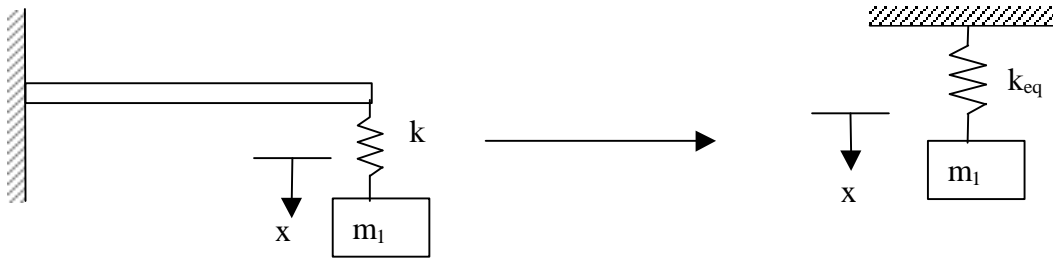
**ES205**  
Examination I  
March 26, 1999

Problem	Score
1	/30
2	/30
3	/40
Total	/100

Show all work for credit  
AND  
Turn in your signed help sheet  
AND  
Stay in your seat until the class ends

1a-1b) The cantilever beam shown below has length,  $L$ , elastic modulus,  $E$ , a mass,  $m_1$ , and a mass moment of inertia,  $I$ . A spring of stiffness,  $k$ , is located at the end of the beam. The spring supports a mass,  $m_2$ . Note:  $x$  is measured from the static equilibrium point.

(1a) (4 pts.) Replace the beam and spring with an equivalent spring system as shown.



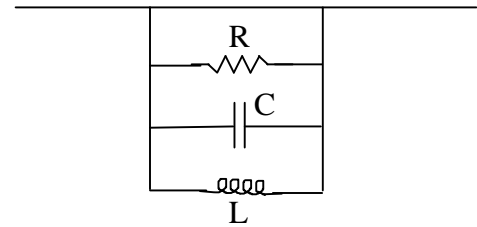
$k_{eq} =$  \_\_\_\_\_

(1b) (2 pts.) What assumption must you make in order to determine  $k_{eq}$ ?

(1c) (4 pts.) A first order system is represented by the following equation:  $\dot{x} + 32x = 64f(t)$   
 What is the time constant of the system?

1d) (4 pts.) Given the differential equation,  $\ddot{x} + 12\dot{x} + 36x = 15\dot{y} + 36y$ , determine the transfer function.  $y$  is the output and  $x$  is the input.

(1e) (4 pts.) Using the complex impedance approach, find  $Z_{eq}$  for the system shown below. Express your final answer as a ratio of simple polynomials.



(1f-1g). The differential equations shown below describe the behavior of a system.  $f(t)$  is the input and  $x_1(t)$  and  $x_2(t)$  are outputs.

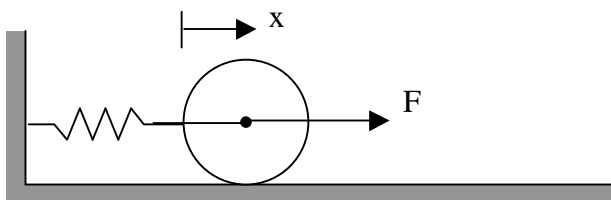
$$m_1 \ddot{x}_1 + b_1 \dot{x}_1 + (k_1 + k_2) x_1 - k_2 x_2 = f(t)$$

$$m_2 \ddot{x}_2 + k_2 \dot{x}_2 - k_2 x_1 = 0$$

(1f) (4 pts.) Represent the system in second order matrix form.

(1g) (4 pts) Determine the state space representation for the system.

(1h) (4 pts) Find the equation of motion for the disk shown below ( $I_G = \frac{1}{2} mr^2$ ) in terms of the output  $x$  and the applied force  $F$ . Assume that the disk rolls without slipping on the ground.



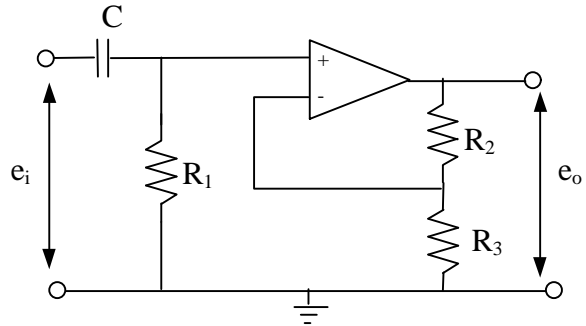
Name \_\_\_\_\_  
ES205 Examination I

**Problem 2**

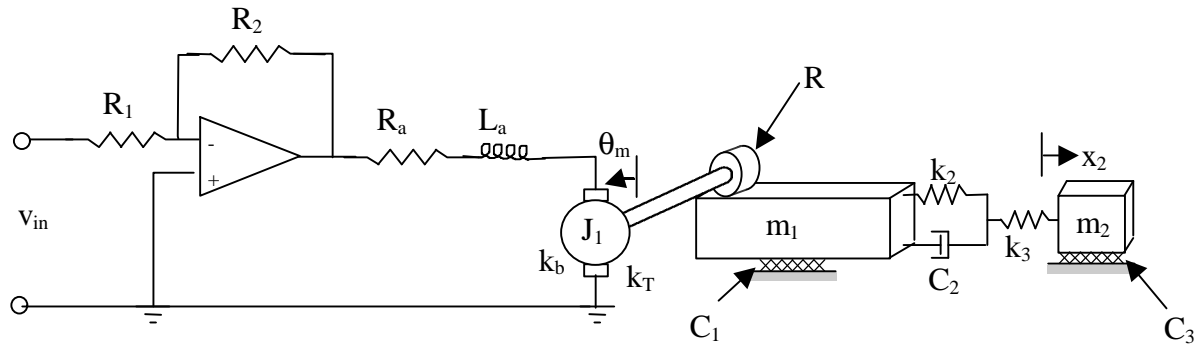
30 pts  
March 26, 1999

An op-amp circuit is shown below. Determine the transfer function between the input  $e_i$  and the output  $e_o$ .

Note: Set up the equations first and save the solution until all the other problems on the test have been solved. Be sure to clearly label all your variables on the circuit diagram.



For the electro-mechanical system shown below determine, in the **Laplace domain**, the necessary equations to find the differential equation of motion (EOM) that relates the known input op-amp voltage,  $v_{in}$ , to the unknown output displacement,  $x_2$ . Clearly define all the unknown variables by labeling them in the picture (some have already been provided to you). Assume the output shaft of the motor is rigid. **Do not find the EOM but number the equations that you would use and generate a list of the unknown variables.**



Unknowns

