

Name \_\_\_\_\_ Section \_\_\_\_\_

**ES205**  
Examination I  
March 27, 1998

Problem	Score
1	/30
2	/30
3	/40
Total	/100

Show all work for credit  
AND  
Turn in your signed help sheet  
AND  
Stay in your seat until the class ends

1a & 1b. A system is found to be governed by the differential equation:

$$\ddot{y} + 3\dot{y} + 2y = 5f(t)$$

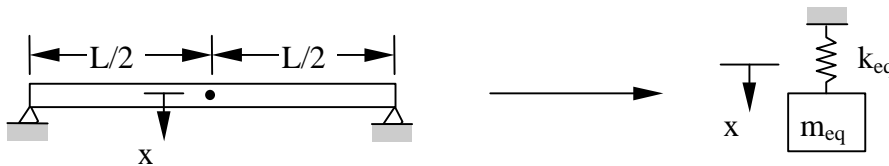
1a) Find the transfer function relating the input  $f$  and the output  $y$

1b) Write  $\ddot{y} + 3\dot{y} + 2y = 5f(t)$  in state space form using  $x_1 = y$ ,  $x_2 = \dot{y}$ ,  $x_3 = \ddot{y}$  by filling in the matrices shown below.

$$\begin{Bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{Bmatrix} = \begin{bmatrix} & & \\ & & \\ & & \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \end{Bmatrix} + \begin{bmatrix} \\ \\ f(t) \end{bmatrix}$$

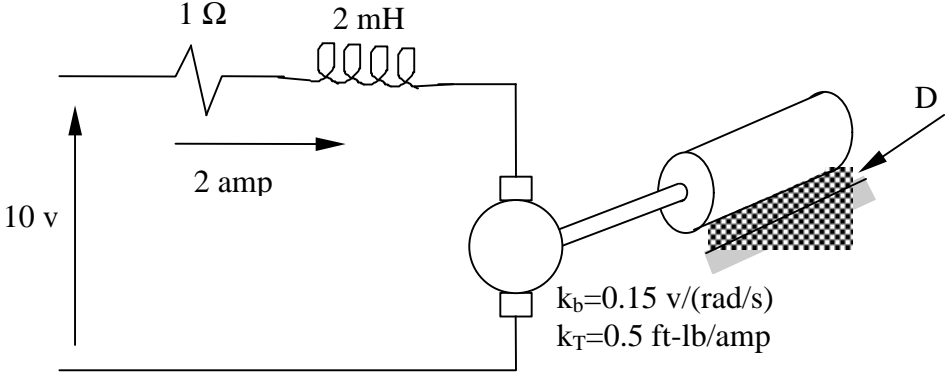
1c) A system is forced by  $f(t) = 5 \sin 3t$  and is found to have a natural frequency of 2 rad/s, damping ratio of 0.2 and static gain of 4. Write the differential equation that describes the system.

1d) For the beam shown below  $EI = 1.6 \times 10^6 \text{ lb-in}^2$  and the length is 20 in and the mass is 1 slug. Estimate the natural frequency of the beam.

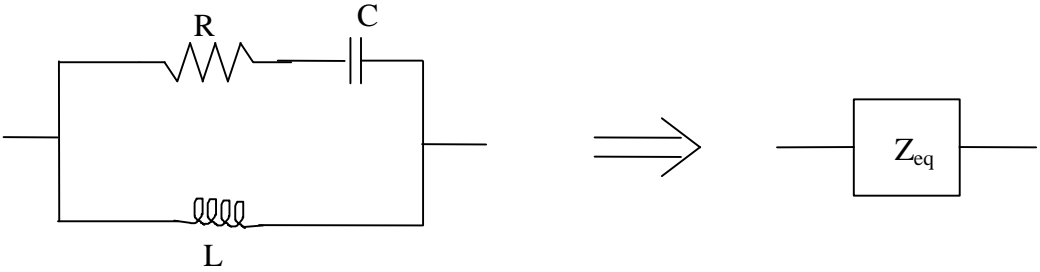


1e and 1f) The permanent magnet DC motor shown below is operating at steady state. The shaft is rotating at 360 rpm. Determine

- 1e) the back emf.
- 1f) the torque produced by the motor

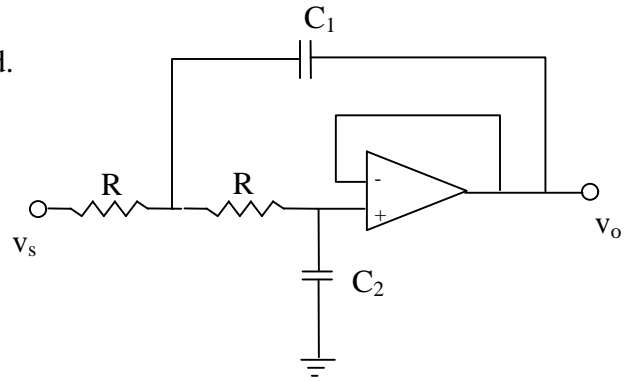


1g) Determine the equivalent complex impedance for the collection of electrical elements shown below. Express your answer as a ratio of simple polynomials.



- A two pole Butterworth low-pass active filter is shown below. Determine
- the transfer function between the input  $v_s$  and the output  $v_o$
  - the natural frequency for this circuit (hint: find the differential equation first)

Note: Set up the equations first and save the solution until all the other problems on the test have been solved.



For the electro-mechanical system shown below determine, in the **Laplace domain**, the necessary equations to find the differential equation of motion (EOM) that relates the known input op-amp voltage,  $v_{in}$ , to the unknown output displacement,  $x$ . Clearly define all the unknown variables by labeling them in the picture. **Do not find the EOM but number the equations that you would use and generate a list of the unknown variables.**

