

Name \_\_\_\_\_ Section \_\_\_\_\_

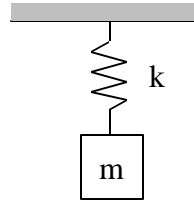
**ES205**  
Examination I  
March 24, 2000

Problem	Score
1	/30
2	/35
3	/35
Total	/100

Show all work for credit  
AND  
Turn in your signed help sheet

**You must show all work for full credit on these problems.**

- 1a) A 1 kg mass is suspended from a spring that has a spring constant of 500 N/m. What is the natural frequency of the system? (4pts)



- 1b) If the spring in problem 1a) has a mass of 0.3 kg what is the percent error in neglecting the mass of the spring in the frequency calculation? (3 pts)

- 1c) A system is represented by the following differential equations:

$$\dot{x} + 3x + 2\dot{y} = 64f(t), \quad \ddot{y} + 0.5\dot{y} + 2y - 4x = 0.$$

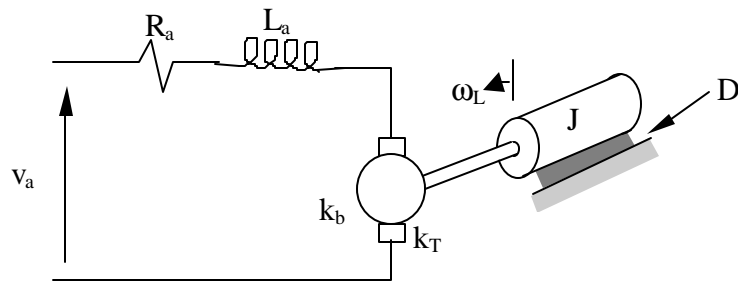
Using the state variables  $x_1 = x$ ,  $x_2 = y$ , and  $x_3 = \dot{y}$  write the state equations for this system by filling in the matrices shown below. (4 pts)

$$\begin{Bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{Bmatrix} = \begin{bmatrix} & & \\ & & \\ & & \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \end{Bmatrix} + \begin{bmatrix} 0 \\ 0 \\ f(t) \end{bmatrix}$$

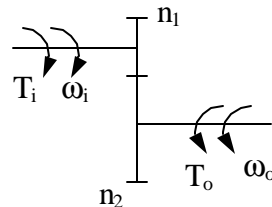
- 1d) The differential equation for a system is given by  $12\ddot{x} + \dot{x} + 3x = 36y$ . Determine the damping ratio of the system. (3 pts)

(1e) (9 pts.)

For the armature controlled DC motor shown below determine the necessary equations to find the differential equation of motion (EOM) that relates the known input voltage,  $v_a$ , to the unknown angular velocity of the load,  $\omega_L$ . Clearly define all the unknown variables by labeling them in the picture (some have already been provided to you).



1f) For the gear system shown below, what is the output torque and angular velocity if  $T_i = 100$  N-m,  $\omega_i = 600$  rad/s. Assume the gears are light (i.e. massless) and the input gear has 15 teeth and the output gear has 45 teeth (4 pts)



1g) The response of a first order system is found to have a time constant equal to 0.5 s and a static gain of 4. What is the differential equation for this system if the input is  $3\sin\omega t$ ? (3pts)

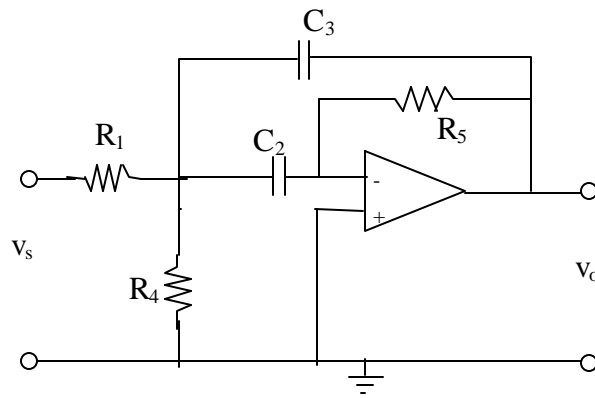
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ES205 Examination I

**Problem 2**

35 pts  
March 24, 2000

A bandpass amplifier op-amp circuit is shown below. Determine the transfer function between the input  $v_i$  and the output  $v_o$ .

Note: Set up the equations first (clearly numbered equations and a list of unknowns) and save the solution until all the other problems on the test have been solved (the actual transfer function is only worth a few points). Be sure to clearly label all your variables on the circuit diagram.



An automobile is modeled as shown below and is currently drawn in a displaced position. The inputs are  $y_1$  and  $y_2$  (assume you know the velocity so  $y_1$  and  $y_2$  are given as functions of time). Derive the equations of motion for outputs  $x_1$ ,  $x_2$  and  $\theta$ . Put these equations in second order matrix form.

Hints: The displacement at the upper end of the springs  $k_1$  will depend on both  $x_1$  and  $\theta$ .  
 Assume small angle so  $\sin\theta = \theta$ .

