Name ____________________ Section (or team) __________

Final Examination

ES205 Analysis and Design of Engineering Systems
Spring 2010

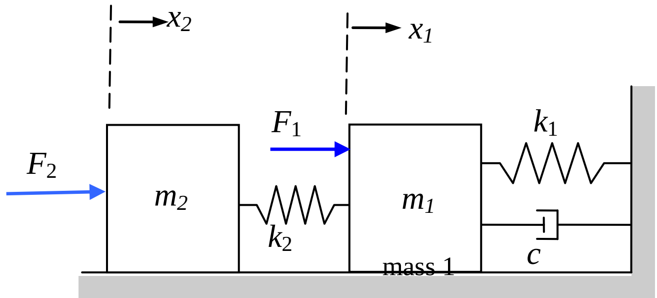
Problem	Answers							Score
1	○ a	○ b	○ c	○ d	○ e	○ f	○ g	/5
2	○ a	○ b	○ c	○ d	○ e	○ f	○ g	/5
3	○ a	○ b	○ c	○ d	○ e	○ f	○ g	/5
4	○ a	○ b	○ c	○ d	○ e	○ f	○ g	/5
5	○ a	○ b	○ c	○ d	○ e	○ f	○ g	/5
6	○ a	○ b	○ c	○ d	○ e	○ f	○ g	/5
7	○ a	○ b	○ c	○ d	○ e	○ f	○ g	/5
8	○ a	○ b	○ c	○ d	○ e	○ f	○ g	/5
9	○ a	○ b	○ c	○ d	○ e	○ f	○ g	/5
10	○ a	○ b	○ c	○ d	○ e	○ f	○ g	/5
							Multiple-choice subtotal:	/50
							11	/10
							12	/10
							13	/10
							14	/30
							15	/20
							16	/30
							Total	/160
							Percentage	/100
							Team incentive	/3
							Exam Score	/100

Instructions

- Seating: at least one empty chair (if possible) between you and your neighbors.
- The exam is closed-book and closed-computer (no laptops for music).
- You may use your calculator.
- No scratch-sheets allowed. All work is to be shown in this test booklet.
- A help-sheet is provided by your instructor. No other help sheets allowed.
- Put your name on this cover page only.
- On multiple choice questions, your work must be shown, correct, and the correct bubble filled on this cover page to receive credit. No partial credit for these problems.
- Partial credit is given for work shown on problems 11–16. Show all your work for maximum credit. Answers without clear support receive little credit.

Problem 1
Determine the equation of motion for mass m_1 of the system shown below. Displacements are measured from the static equilibrium position (SEP).

a) $20(\ddot{x}_1 - \ddot{x}_2) + 5(\dot{x}_1 - \dot{x}_2) + 20(x_1 - x_2) = F_1 - F_2$
b) $20\ddot{x}_1 + 5\dot{x}_1 + 30x_1 = F_1$
c) $20\ddot{x}_1 + 5\dot{x}_1 + 30x_1 - 10x_2 = F_1$
d) $20\ddot{x}_1 + 5\dot{x}_1 + 10x_1 - 10x_2 = F_1$
e) $20\ddot{x}_1 + 5\dot{x}_1 + 10x_1 + 10x_2 = F_1$
f) none of the above



$k_1 = 20$ N/m
$k_2 = 10$ N/m
$m_1 = 20$ kg
$c = 5$ N/(m/s)

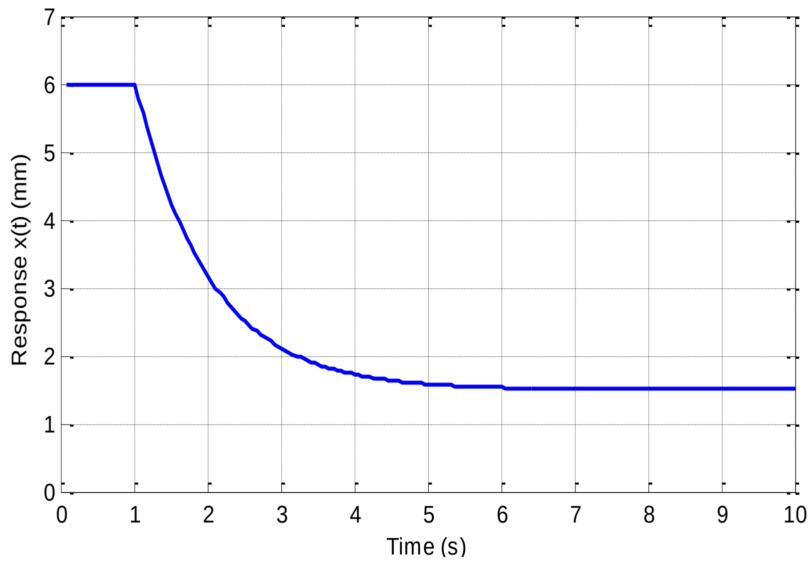
Assume the surface is frictionless

Problem 2
A second-order system has a damping ratio of 0.2, a static gain of 3, and an observed period of oscillation of 1.28 s. The system model is most nearly:

a) $0.040\ddot{z} + 0.080\dot{z} + z = 3f(t)$
b) $0.042\ddot{z} + 0.082\dot{z} + z = 3f(t)$
c) $25\ddot{z} + 2\dot{z} + z = 3f(t)$
d) $\ddot{z} + 2\dot{z} + 25z = 3f(t)$
e) $\ddot{z} + 0.08\dot{z} + 0.04z = 3f(t)$
f) $0.01\ddot{z} + 0.08\dot{z} + z = 3f(t)$
g) none of the above

Problem 3
A linear 1st-order system has the input $f(t)$ and the response $x(t)$ shown below.

$$f(t) = \begin{cases} 2 & -\infty < t < 1 \\ 1/2 & t > 1 \end{cases}$$



Choose the best estimate of the system model.

a) $\dot{x} + x = 0.75f$

b) $\dot{x} + x = 3f$

c) $1.65\dot{x} + x = \frac{3}{2}f$

d) $1.65\dot{x} + x = 3f$

e) $1.65\dot{x} + x = -3f$

f) $2\dot{x} + x = 3f$

g) $2.84\dot{x} + x = \frac{3}{2}f$

Problem 4
A chip with a surface area of 25 mm^2 and emissivity of 0.8 is cooled by a fluid with a convective heat transfer coefficient h. The temperature of the fluid around the chip is 25°C and the temperature of the surroundings is 30°C. When the chip power is 0.10 W, the steady state temperature of the chip is 40°C. Determine the amount of heat transfer due to radiation. Note: $\sigma = 5.67\times10^{-8}$ W/(m^{2}K^4). Select the closest answer.

a) 0.00198 mW
b) 1.33 W
c) 1.98 mW
d) 0.0113 mW
e) 1.33 mW

Problem 5

Given:	$\ddot{x} + 11\dot{x} + 24x = 12u(t)$	A second-order system subjected to a step input.
	$x_h(t) = C_1 e^{-3t} + C_2 e^{-8t}$	The homogeneous (or complementary) solution.
	$x(0) = 0; \qquad \dot{x}(0) = 2$	Initial conditions.

Find the equations used to determine C_1 and C_2. Do not solve.

a) $$\begin{aligned} 0 &= C_1 + C_2 \\ 2 &= -3C_1 - 8C_2 \end{aligned}$$

b) $$\begin{aligned} -2 &= C_1 + C_2 \\ 2 &= -3C_1 - 8C_2 \end{aligned}$$

c) $$\begin{aligned} -\frac{1}{2} &= C_1 + C_2 \\ 2 &= -3C_1 - 8C_2 \end{aligned}$$

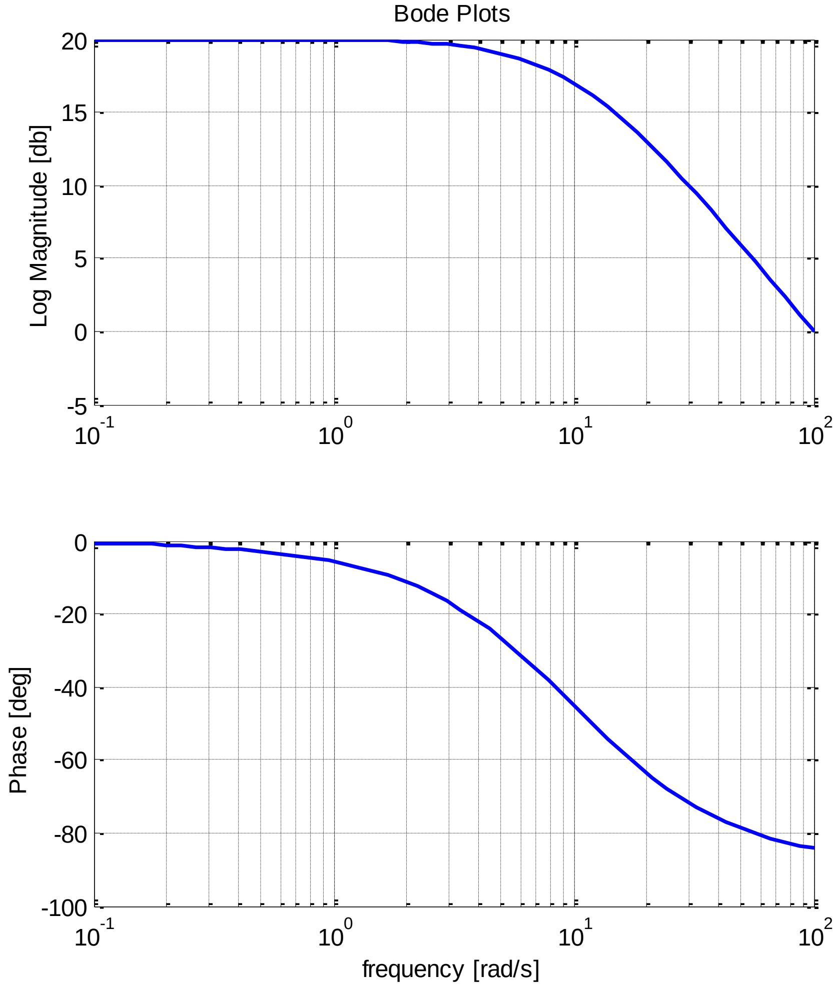
d) $$\begin{aligned} -1 &= C_1 + C_2 \\ 2 &= -3C_1 - 8C_2 \end{aligned}$$

e) $$\begin{aligned} -\frac{1}{2} &= C_1 + C_2 \\ 2 &= 3C_1 + 8C_2 \end{aligned}$$

Problem 6
A system's frequency response plot is given.

Determine the system response to a unit step input with zero initial conditions.

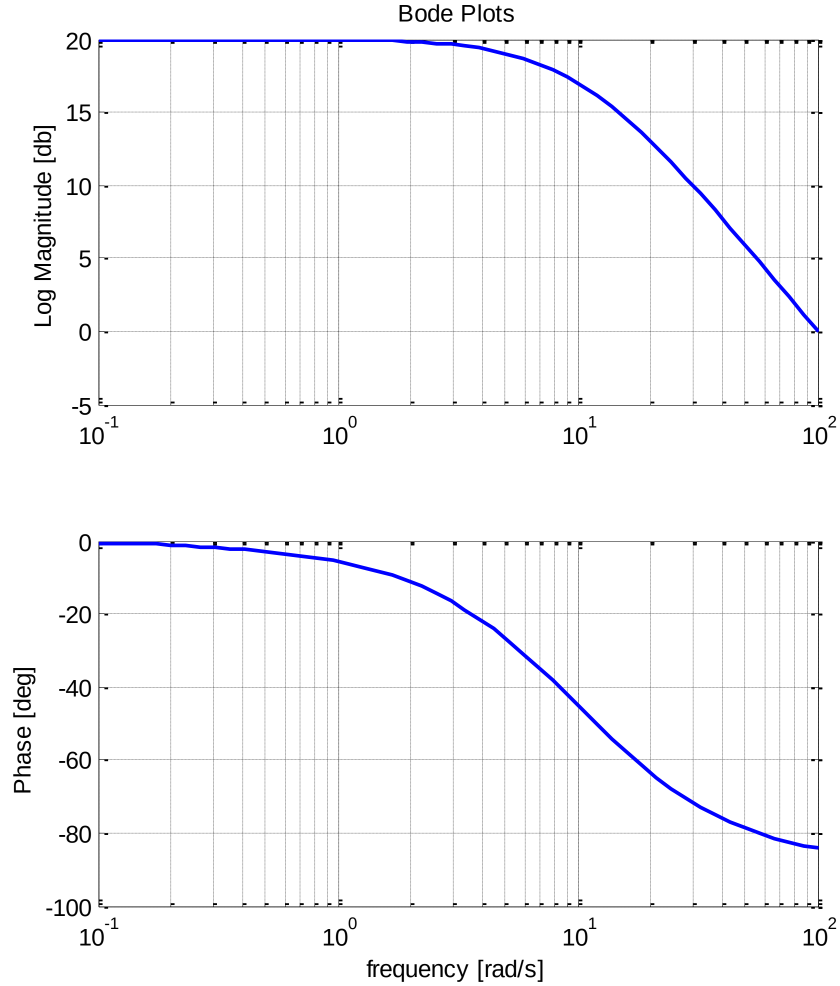
a) $10(1-e^{-10t})$
b) $1-e^{-t/10}$
c) $20(1-e^{-10t})$
d) $20e^{-t/10}$
e) $10\left(1-e^{-t/10}\right)$



Problem 7
A system's frequency response plot is given.

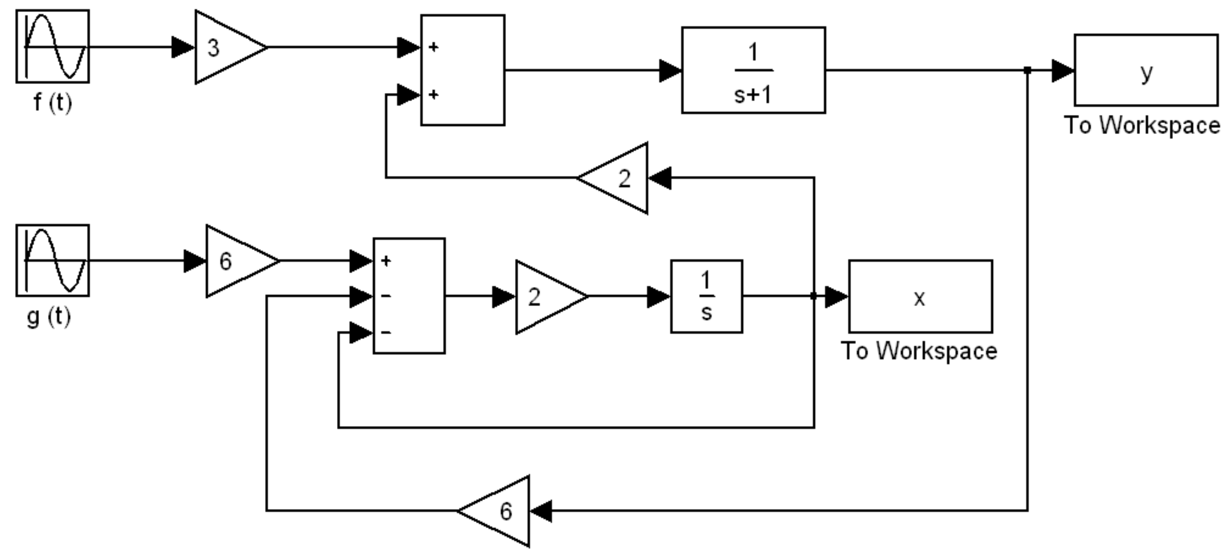
Determine the steady-state output to the input:

$f(t) = 5\cos(12t + 15^o)$.



a) $16 \cos(12t - 55^o)$
b) $32 \cos(12t - 35^o)$
c) $32 \cos(12t - 70^o)$
d) $50 \cos(12t - 50^o)$
e) $50 \cos(12t - 70^o)$
f) $80 \cos(12t - 35^o)$
g) $80 \cos(12t - 70^o)$

Problem 8
Identify a state space matrix that is consistent with the simulation diagram shown below.



a) $\begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} = \begin{bmatrix} -1 & 6 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 3 & 0 \\ 0 & 6 \end{bmatrix} \begin{bmatrix} f(t) \\ g(t) \end{bmatrix}$

b) $\begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} = \begin{bmatrix} -1 & 3 \\ 1 & -6 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 3 & 0 \\ 0 & 6 \end{bmatrix} \begin{bmatrix} f(t) \\ g(t) \end{bmatrix}$

c) $\begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} = \begin{bmatrix} -0.5 & 6 \\ 2 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 0 & 6 \\ 3 & 0 \end{bmatrix} \begin{bmatrix} f(t) \\ g(t) \end{bmatrix}$

d) $\begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} = \begin{bmatrix} -1 & 6 \\ 2 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 0 & 12 \\ 3 & 0 \end{bmatrix} \begin{bmatrix} f(t) \\ g(t) \end{bmatrix}$

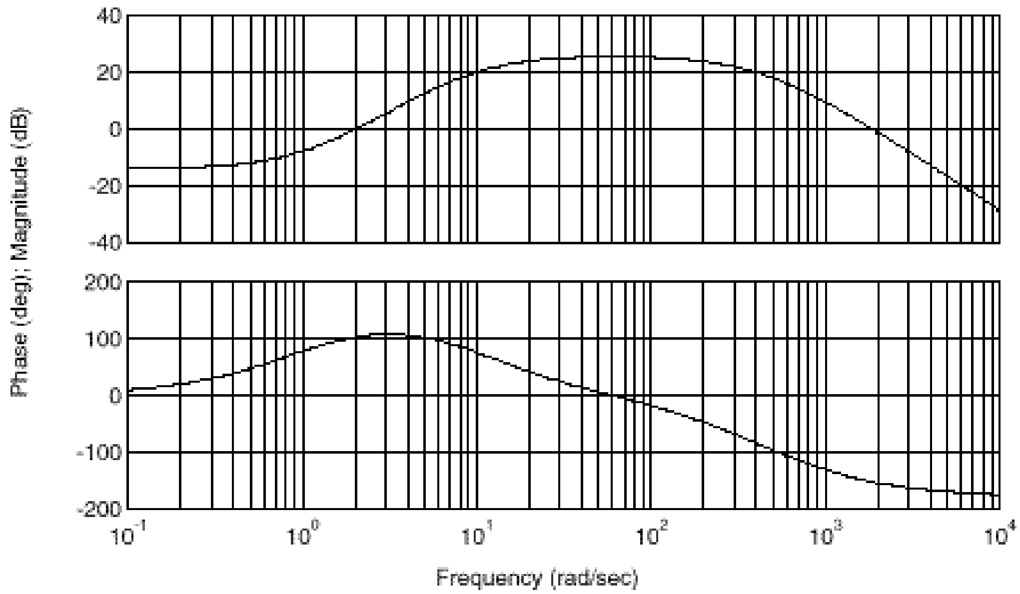
e) $\begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} = \begin{bmatrix} -2 & -12 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 0 & 12 \\ 3 & 0 \end{bmatrix} \begin{bmatrix} f(t) \\ g(t) \end{bmatrix}$

f) $\begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} = \begin{bmatrix} -2 & -12 \\ 2 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 0 & 12 \\ 3 & 0 \end{bmatrix} \begin{bmatrix} f(t) \\ g(t) \end{bmatrix}$

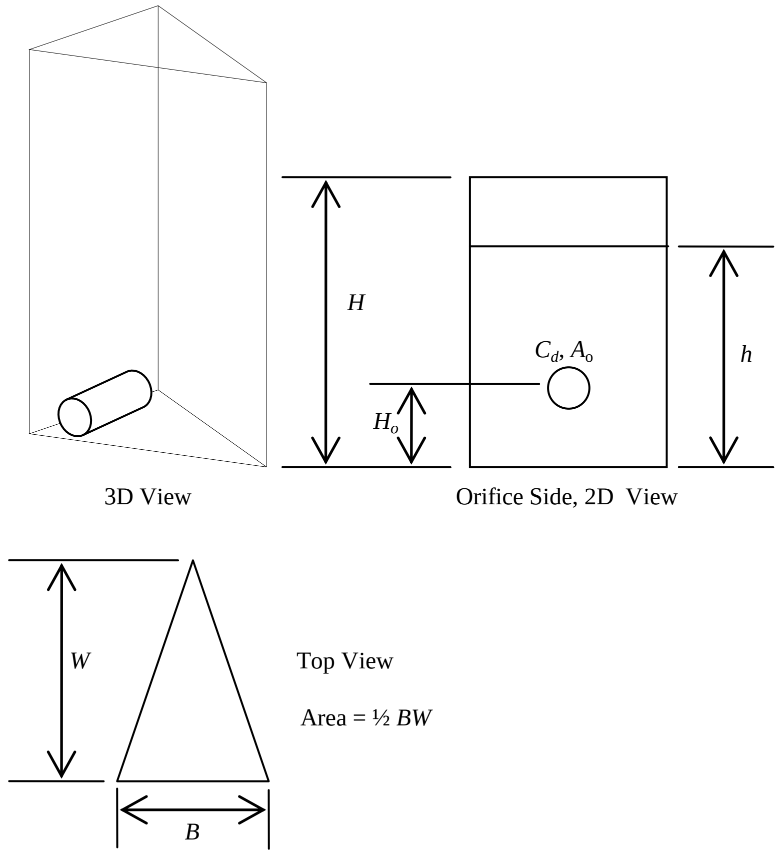
g) none of the above

Problem 9
A frequency response plot for a system is shown. If the system input is $f(t) = \cos(\omega t)$, over what range of input frequencies will the output be greater than the input?

a) No frequencies
b) 0.2 rad/s < ω < 4000 rad/s
c) 1 rad/s < ω < 1000 rad/s
d) 2 rad/s < ω < 2000 rad/s
e) 10 rad/s < ω < 400 rad/s
f) All frequencies



Problem 10
A tank with a triangular cross-section and straight sides has an orifice on one side. at height H_o above the bottom of the tank. The height of the tank is H, and the fluid level in the tank is h. Model the tank fluid height in terms of h.



a) $\pi W^2 \frac{dh}{dt} = -C_d A_0 \sqrt{2g(h - H_0)}$

b) $HW \frac{dh}{dt} = -C_d A_0 \sqrt{2g(h - H_0)}$

c) $BW \frac{dh}{dt} = C_d A_0 \sqrt{2g(h - H_0)}$

d) $BW\left(\frac{dh}{dt} + \frac{dH_0}{dt}\right) = -C_d A_0 \sqrt{2gh}$

e) $\frac{1}{2} BW \frac{dh}{dt} = -C_d A_0 \sqrt{2g(h + H_0)}$

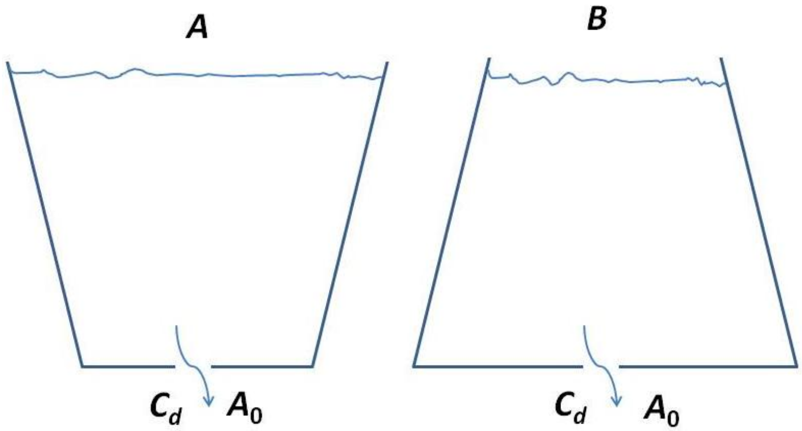
f) $\frac{1}{2} BW\left(\frac{dh}{dt} + \frac{dH_0}{dt}\right) = -C_d A_0 \sqrt{2gh}$

g) $\frac{1}{2} BW \frac{dh}{dt} = -C_d A_0 \sqrt{2g(h - H_0)}$

Fill in the bubbles with the appropriate answers on the exam cover page.
This step is mandatory for credit on the multiple-choice questions.

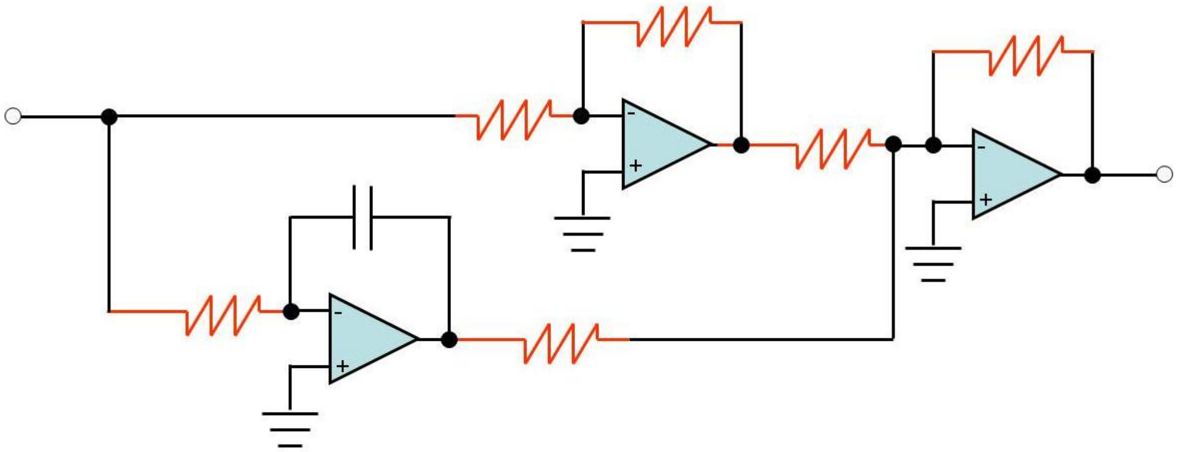
Problem 11

[10 points] Two tanks *A* and *B* start with the same volume of water at the same height. Both are open to the atmosphere, with an orifice at the bottom having the same discharge coefficient and orifice area.



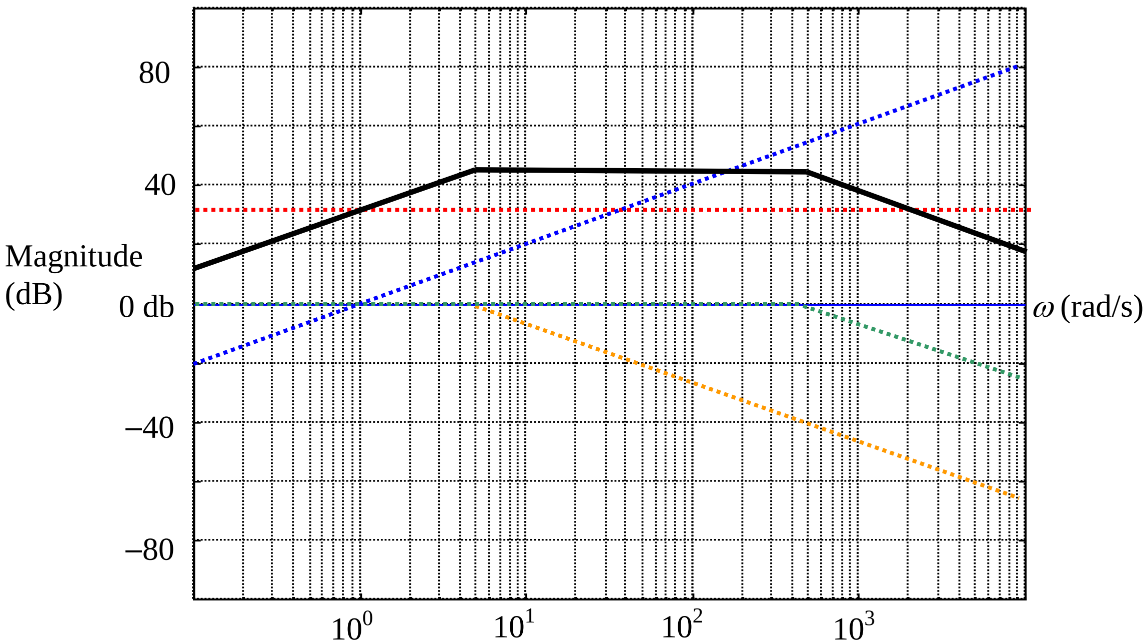
Explain which tank empties soonest and why.

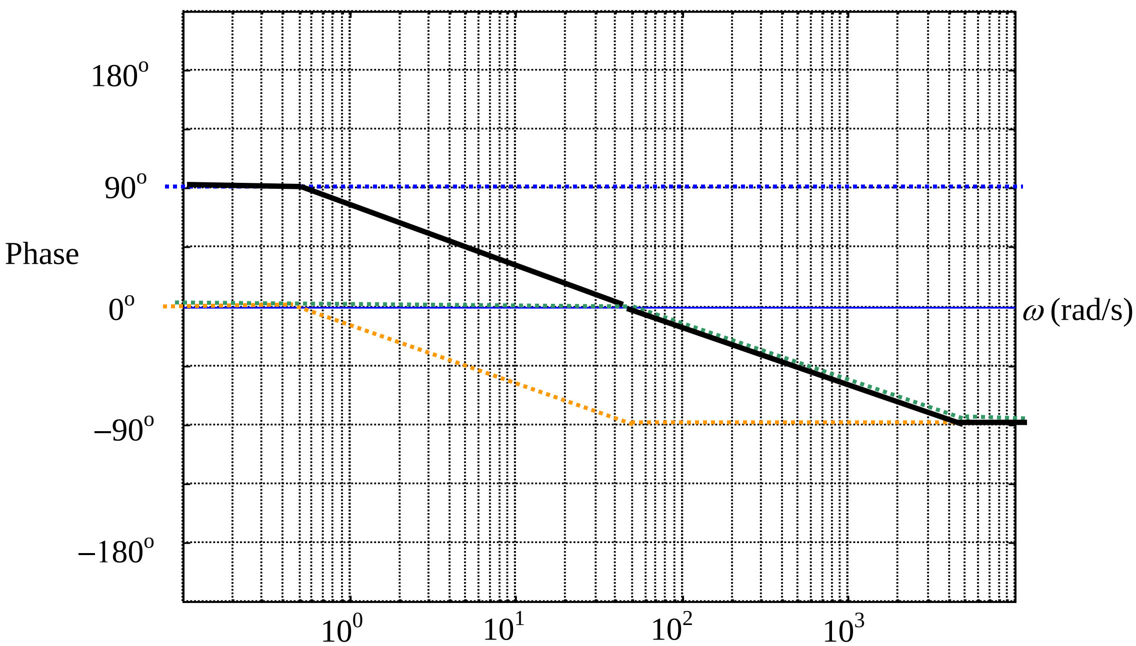
Problem 12
[10 points] Obtain a transfer function relating the output voltage to the input voltage.



[3 points] *Extra credit*. This circuit provides what functionality to a system?

Problem 13
[10 points] Determine the transfer function represented by the asymptotic frequency-response shown.





Problem 14

[30 points] A tape-transport system controls the tension and velocity of a moving tape.

- The tape is modeled as rigid, massless sections connected by springs.
- A force P_1 is applied by a vacuum chamber (not shown) to keep the tape in tension.
- A reaction force P_2 is applied by supply reel (not shown).
- The armature-controlled DC motor imparts the motion to the tape.
- The subscripted variables, e_1, e_2, and e_a represent nodal voltages with respect to ground.
- Assume displacements are measured from the static equilibrium position (SEP).
- For the tachometer assume the output voltage is equal to $k_p \times \dot{\theta}_1$.
- The parameters J_1 and J_2 are combined inertias for the pulley and motor systems.
- Inputs P_1, P_2, and e_r and the system parameters are known

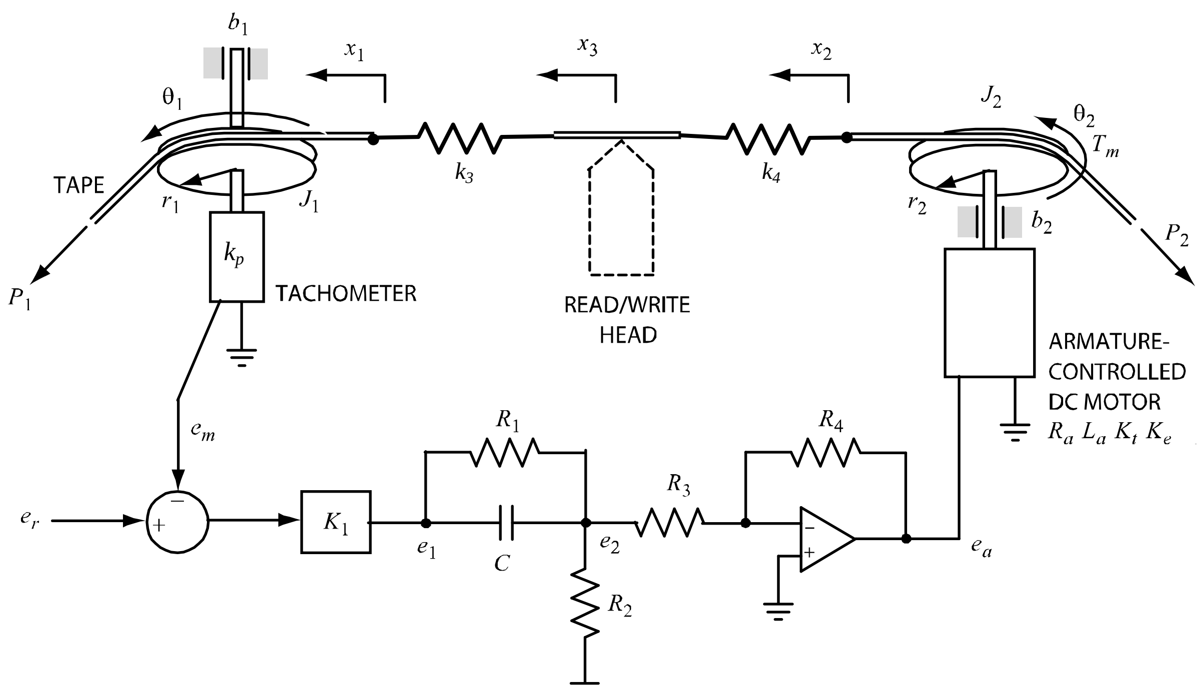
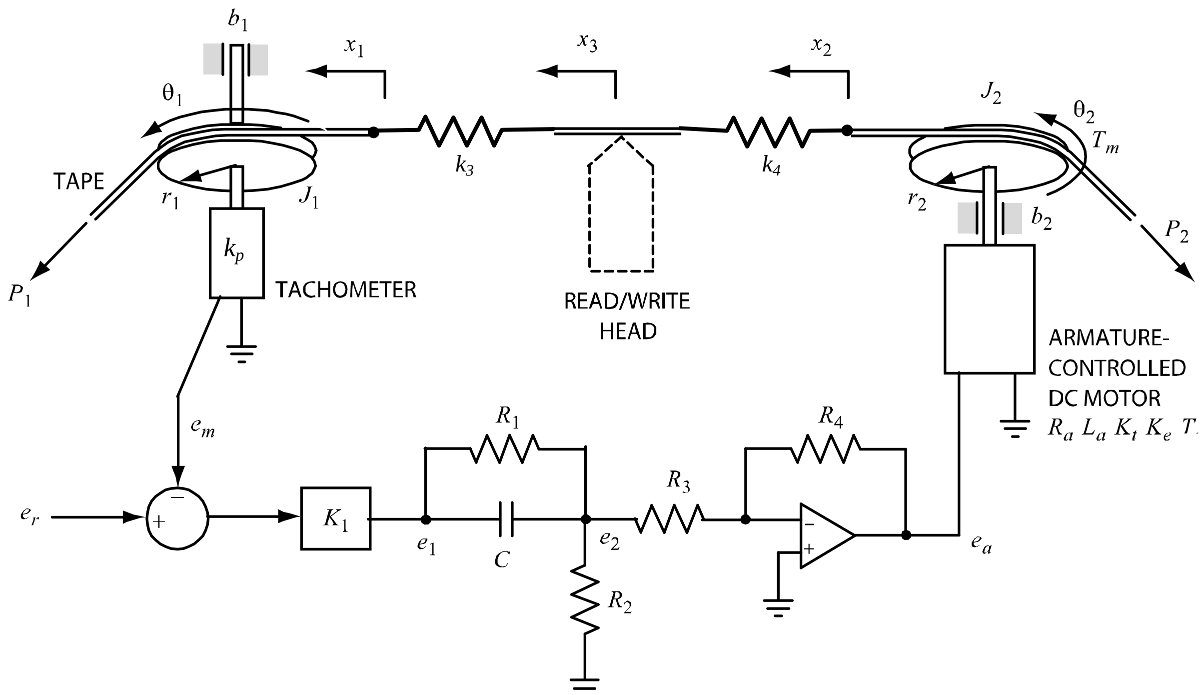


Fig. 14: Tape transport system.

Model the system, including

- drawing appropriate sketches
- stating appropriate principles
- numbering the equations
- listing the unknowns

(*Go to the next page for space to show your work.*)



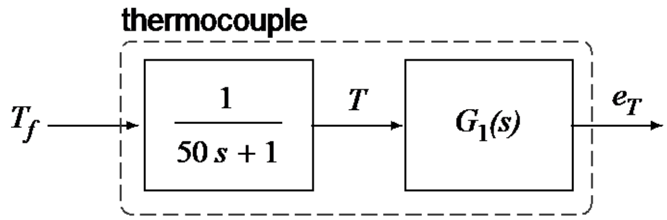
<u>Unknowns</u>

Continue on the next page if needed.

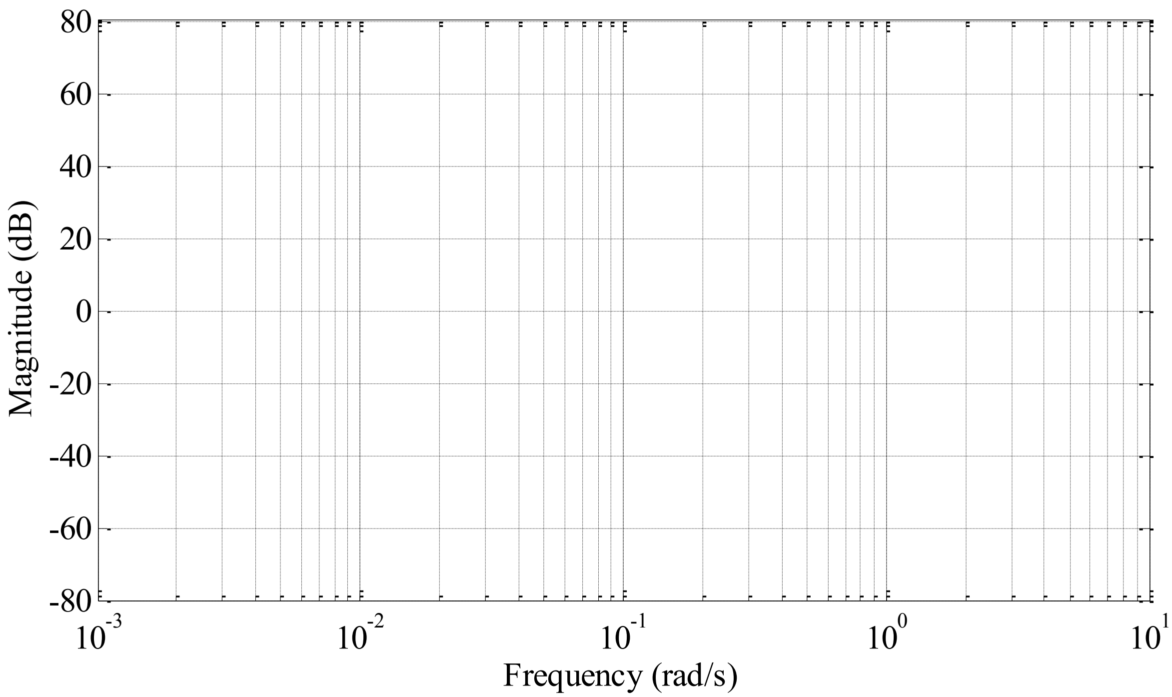
...continue your work here if needed...

Problem 15
[20 points] We measure a periodically-varying fluid-temperature T_f using a thermocouple. Assume that all temperature measurements are in ˚F and *do not* have to be converted to Rankine.

15.1 [10 points] The thermocouple assembly is modeled using a first-order transfer function with output temperature T in series with a gain G_1 that converts ˚F to the output voltage e_T. The constant gain is G_1 = 10mV/˚F. (Leaving the units in of e_T in mV, the gain G_1 is 10.)

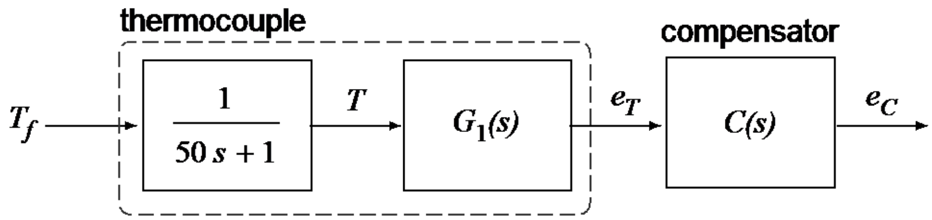


On the semilog graph paper provided below, use a straightedge to sketch the straight-line asymptotic approximate frequency response (magnitude only) of the transfer function relating e_T to T_f . Show your work.

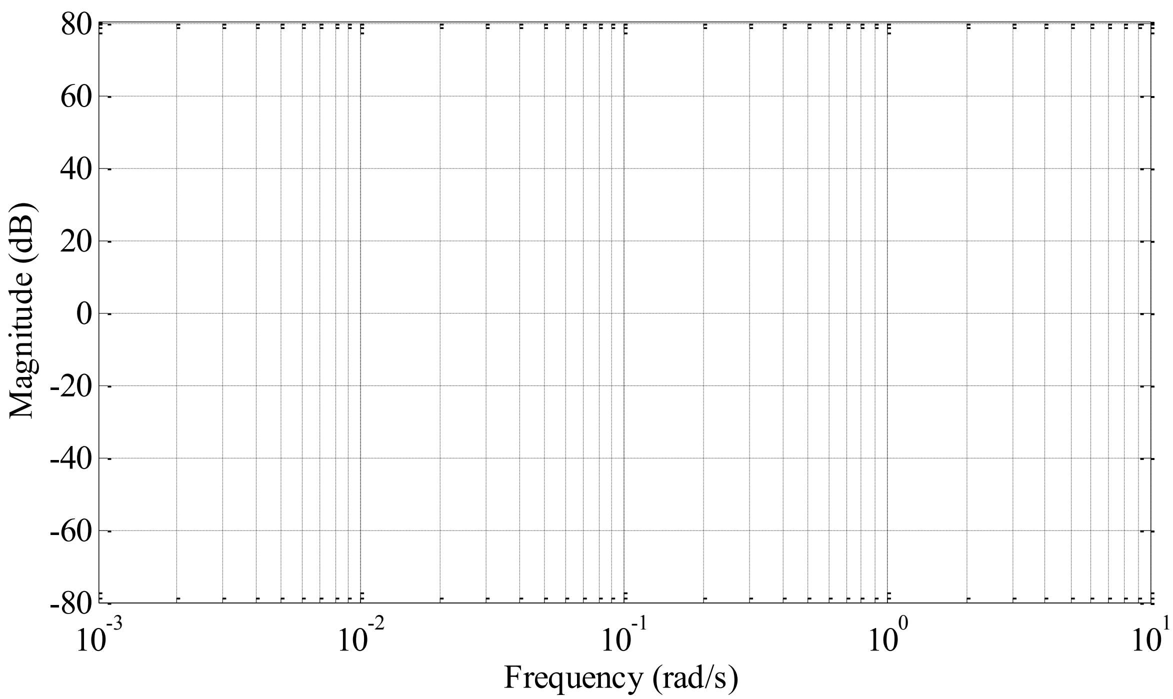


15.2 [10 points] Assume that the compensator has the transfer function

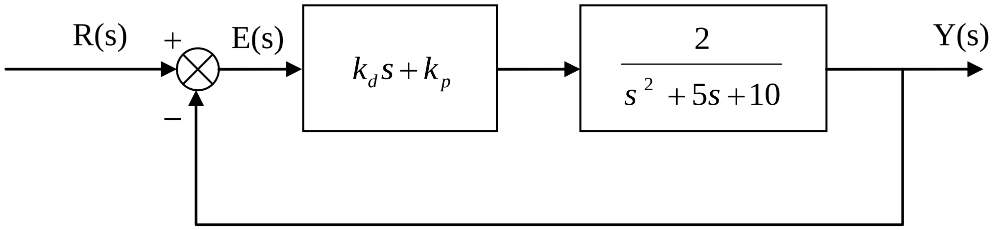
$$C(s) = \frac{50s + 1}{2000\,s + 1000}$$



Recall that gain G_1 = 10. On the semilog axis below, sketch the new frequency response (magnitude only) of the transfer function relating e_C to T_f . Show your work.



Problem 16
[30 points] A PD controller is shown below for a second-order plant. The proportional gain is k_p and the derivative gain is k_d, with $k_p > 0$ and $k_d > 0$.



16.1 [10 points] Find the closed-loop transfer function. Arrange terms so that both numerator and denominator are polynomials.

16.2 [15 points] Assume that the answer to 16.1 is given by

$$\frac{Y}{R}(s)=\frac{10k_D s+\frac{5000}{k_P^2}}{s^2+5k_D s+\frac{2500}{k_P^2}}$$

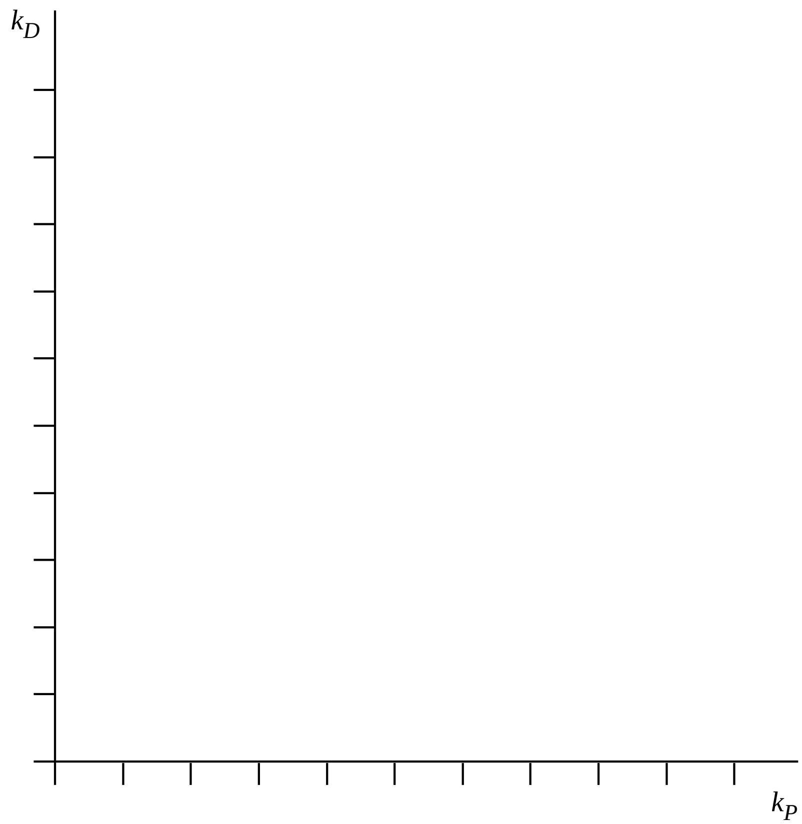
This is not the correct answer to 16.1. Just assume it is for this part of the problem.

Given the following performance specifications,

- $\omega_n \leq 10$ rad/s
- overshoot $\leq 44.4\%$

Find the design limits in terms of k_p and k_d.

16.3 [5 points] Sketch and label the design region on the (k_p, k_d) axes. Label the axis scales.



End of the Final Exam