

Review of Mechanical Systems

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Kinetics

1) Conservation of linear momentum and angular momentum rate form

Principle

Linear Momentum:

$$\frac{d\vec{P}_{sys}}{dt} = \sum \vec{F} + \sum_{in} \dot{m}_i \vec{v}_i - \sum_{out} \dot{m}_o \vec{v}_o$$

Angular Momentum about point O:

$$\frac{d\vec{L}_{sys_o}}{dt} = \sum \vec{M}_o + \sum_{in} \vec{r} \times \dot{m}_i \vec{v}_i - \sum_{out} \vec{r} \times \dot{m}_o \vec{v}_o$$

For this class we usually looked at closed systems so $\frac{d\vec{P}_{sys}}{dt} = \sum \vec{F}$ and $\frac{d\vec{L}_{sys_o}}{dt} = \sum \vec{M}_o$

Procedure

1. Identify the system. The system you pick usually involves the forces and or accelerations in the find statement.
2. Draw the free body diagram (FBD). Include all external forces and moments. Don't forget gravity.
3. Draw the kinetic diagram (KD). For plane motion be sure to include $m\vec{a}_G$ and $I_G\alpha$. The constraints on the problem may permit you to limit the direction of \vec{a} . For example:
 - Translation ($\alpha = 0$): Put \vec{a} in the known direction recognizing that the acceleration of the center of gravity is equal to the acceleration of any other point on the body.
 - Fixed axis rotation: Put tangential acceleration (αr) and normal acceleration ($\omega^2 r$) in known directions. Be sure $I_G\alpha$ and $m(\alpha r)$ are consistent with each other.
 - General plane motion (rolling without slipping, CG = geometric center): $a_G = \alpha r$. Be sure $I_G\alpha$ and $m(\alpha r)$ are consistent with each other.
 - General plane motion: Put ma_{G_x} , ma_{G_y} and $I_G\alpha$ on the KD. Use kinematics to relate the acceleration of the center of gravity to the acceleration of some point you know something about.
4. Place a coordinate system between the FBD and KD to be used when applying conservation of linear momentum.
5. Apply the scalar equations for conservation of linear and angular momentum to the FBD and KD. Be sure to define a positive direction for the summations.

$$\frac{dP_{sys_x}}{dt} = \sum F_x, \quad \frac{dP_{sys_y}}{dt} = \sum F_y, \quad \frac{dL_{sys_o}}{dt} = \sum \vec{M}_o$$

6. Solve the equations for the required unknowns. If the three scalar equations have more than three unknowns then a kinematic analysis may be required for the problem solution. If you still need more equations after a kinematic analysis then try another FBD/KD of an interrelated system.

Additional notes on kinetics:

Rolling motion (for an object that has a radius r and a center of gravity located at the geometric center)

Rolling, no sliding:	$a_G = \alpha r, F = ?$ (but it is $< \mu_s N$)
Rolling, sliding impending:	$a_G = \alpha r, F = \mu_s N$
Rotating and sliding:	$a_G = ?, \alpha = ?, F = \mu_k N$ ($a_G \neq \alpha r$)

2) Conservation of energy finite time form (for an adiabatic, closed system)

Principle

$$\Delta E_{sys} = W$$

where:

W = work of all external forces and moments

for a force: $W = \int_1^2 \vec{F} \cdot d\vec{r}$

for a moment: $W = \int_1^2 M d\theta$

for rolling without slipping a fixed surface: $W_{friction} = 0$

$E_{sys} = E_k + E_G + E_s + U \dots$

E_k = kinetic energy = $\frac{1}{2}mv_G^2 + \frac{1}{2}I_G\omega^2$ for each rigid body in the system

note: there will always be a kinematic step of relating v_G to ω .

E_G = gravitational potential energy = mgy for each rigid body in the system

note: y is measured from the datum to the center of gravity

E_s = elastic potential energy for a linear spring = $\frac{1}{2}kx^2$ for each spring in the system

note: x is measured from the free length of the spring

Procedure

1. Identify the system.
2. Draw diagrams of the system in position 1 and position 2.
3. Locate a datum (if any center of gravity changes elevation)
4. Calculate each term in the energy equation.

3) Conservation of linear momentum and angular momentum finite time form (closed system)

Principle

$$\Delta \vec{P}_{sys} = \int_{t_1}^{t_2} \vec{F} dt$$

$$\Delta \vec{L}_{sys_0} = \int_{t_1}^{t_2} \vec{M}_0 dt$$

or if there are impulsive loads acting on the system

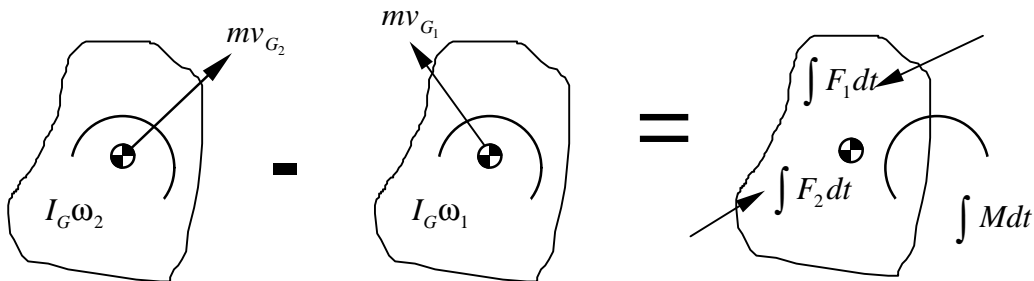
$$\Delta \vec{P}_{sys} = \sum \vec{F}_i \Delta t$$

$$\Delta \vec{L}_{sys_0} = \sum (\vec{M}_0)_i \Delta t$$

It is important to know that these are vector equations so just like the rate form we can obtain scalar equations by looking at the linear momentum in the x and y directions and the angular momentum about some point.

Procedure

1. Identify the system.
2. Draw impulse-momentum diagrams



System momentum after the time interval

System momentum before the time interval

Impulses acting during the time interval

3. Define a coordinate system to be used when applying conservation of linear momentum.
4. Apply the scalar equations for conservation of linear and angular momentum to impulse-momentum diagrams. We therefore have three scalar equations
 1. Linear momentum in the x-direction
 2. Linear momentum in the y-direction
 3. Angular momentum (moment of the momentum) about any axis
5. Solve the equations for the required unknowns. If the three scalar equations have more than three unknowns then a kinematic analysis may be required for the problem solution. If you have an impact you may need to use the coefficient of restitution equation (if e is given or one of the unknowns you are to solve for)

Notes:

- If there are impulsive forces acting in the system you can neglect non-impulsive forces.
- This principle is useful for central force problems.
- The coefficient of restitution is required for some impact problems.
- Additional comments on impact problems:

Procedure for impact problems:

1. Draw the impact (impulse-momentum diagrams are especially helpful for rigid body impact).
2. Look at each object by itself, that is, define your system to be each object individually first.
 - Is there any direction with no impulsive forces acting? If so, apply LM in that direction.
Note: A direction that frequently does not have any impulses acting is along the plane of contact.
 - Is there any point that you could take angular momentum about that has no impulsive moments? If so, take apply AM about that point.
3. Look at both objects together, that is, define your system to be both objects.
 - Is there any new direction with no impulsive forces acting? If so, apply LM in that direction.
 - Is there any point that you could take angular momentum about that has no impulsive moments? If so, take apply AM about that point.
4. Coefficient of restitution (experimentally determined for a given combination of materials, assumed to be constant)

$$e = -\left(\frac{v'_{PB_n} - v'_{PA_n}}{v_{PB_n} - v_{PA_n}} \right) \quad \text{or} \quad v'_{PB_n} - v'_{PA_n} = e(v_{PA_n} - v_{PB_n})$$

where

e = the coefficient of restitution

v'_{PA_n} = the velocity of the point of contact on object A in the normal direction after the impact

v'_{PB_n} = the velocity of the point of contact on object B in the normal direction after the impact

v_{PA_n} = the velocity of the point of contact on object A in the normal direction before the impact

v_{PB_n} = the velocity of the point of contact on object B in the normal direction before the impact

