Kinematics (the geometry of the motion)

Basic kinematic relationships

For a point	For a rigid body
$v = \frac{dx}{dt} = \dot{x}$	$\omega = \frac{d\theta}{dt} = \dot{\theta}$
$a = \frac{dv}{dt} = \frac{d^2x}{dt^2} = \ddot{x}$ or $a = v\frac{dv}{dx}$ (From using the chain rule)	$\alpha = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2} = \ddot{\theta}$ or $\alpha = \omega\frac{d\omega}{d\theta}$ (From using the chain rule)

To obtain algebraic relationships separate variables and integrate. For example:

Constant acceleration equations (useful for any problem with constant acceleration - such as projectile motion):

a = constant	α = constant
$x = x_0 + v_0 t + \frac{1}{2}at^2$	$\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$
$v = v_0 + at$	$\omega = \omega_0 + \alpha t$
$v^2 = v_0^2 + 2a(x - x_0)$	$\omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0)$

We can also express the velocity and acceleration in various coordinate systems. This is helpful when we apply linear momentum.

Rectangular components

$$\vec{v} = v_x\hat{\mathrm{i}} + v_y\hat{\mathrm{j}} \qquad \vec{a} = a_x\hat{\mathrm{i}} + a_y\hat{\mathrm{j}}$$

Normal and tangential components

$$\vec{v} = v\hat{\mathrm{e}}_{\mathrm{t}} \qquad \vec{a} = \dot{v}\hat{\mathrm{e}}_{\mathrm{t}} + \frac{v^2}{\rho}\hat{\mathrm{e}}_{\mathrm{n}}$$

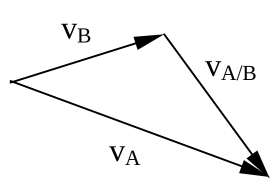
Radial and transverse components

$$\vec{v} = \dot{r}\hat{e}_r + r\dot{\theta}\hat{e}_\theta \qquad \vec{a} = \left(\ddot{r} - r\dot{\theta}^2\right)\hat{e}_r - \left(r\ddot{\theta} + 2\dot{r}\dot{\theta}\right)\hat{e}_\theta$$

Other topics:

Relative Motion

$$\vec{v}_A = \vec{v}_B + \vec{v}_{A/B} \qquad \vec{a}_A = \vec{a}_B + \vec{a}_{A/B}$$ (Vector Equations)



Dependent Motion

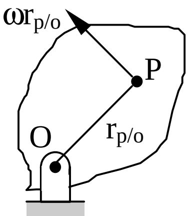
1) Define coordinates, 2) write constraint equations, 3) differentiate

Rigid Body Kinematics

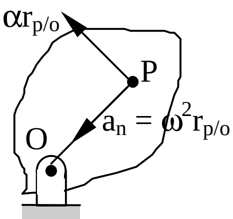
Translation - All point on the body have the same velocity and acceleration

Fixed axis rotation (i.e. find the velocity and acceleration of any point on the rigid body)

velocity: magnitude = ωr
direction = perpendicular to r



acceleration: tangential component = αr
normal component = $\omega^2 r$



General plane motion (always valid for two points on the same rigid body)

$$\vec{v}_A = \vec{v}_B + \vec{v}_{A/B}$$
$$= \vec{v}_B + \vec{\omega} \times \vec{r}_{A/B}$$

$$\vec{a}_A = \vec{a}_B + \vec{a}_{A/B}$$
$$= \vec{a}_B + \vec{\alpha} \times \vec{r}_{A/B} + \vec{\omega} \times (\vec{\omega} \times \vec{r}_{A/B})$$
$$= \vec{a}_B + \vec{\alpha} \times \vec{r}_{A/B} - \omega^2 \vec{r}_{A/B} \quad \text{(plane motion)}$$

To solve the velocity problem it is often easiest to instantaneous centers. **Instantaneous center cannot be used for accelerations**!

Instantaneous Center (draw lines perpendicular to velocity)