OBJECTIVES -- Conservation of Angular Momentum

1. Define, explain, compare and contrast the following terms and concepts:

Angular position: q - radians
Angular velocity: w - radians/second
Angular acceleration: a - radians/(seconds)^2

Angular momentum about origin *O*

angular momentum about the origin *O* for a particle: $\vec{L}_o = \vec{r} \times m\vec{V}$

specific angular momentum about the origin *O* : $\vec{r} \times \vec{V}$

where $\vec{r}$ is the position vector with respect to the origin *O*

right-hand rule sign convention
vector nature of angular momentum
units of angular momentum (N·m·s, lbf·ft·s)

Application of Accounting Principle for Angular Momentum

rate of accumulation of angular momentum with the system

amount of angular momentum about origin *O*: $\vec{L}_{o,sys} = \iiint_V (\vec{r} \times \vec{V})\ r\ dV$

mass moment of inertia about a single axis: $I = \iiint r^2\ r\ dV$

relation between mass moment of inertia, angular momentum, and angular velocity

transport rate of angular momentum across system boundaries

torques or moments produced by external forces about origin *O*: $\sum \vec{M}_o = \sum \vec{r} \times \vec{F}_{ext}$

mass transport of angular momentum about the *O*: $\sum \dot{m}(\vec{r} \times \vec{V})_{in} - \sum \dot{m}(\vec{r} \times \vec{V})_{out}$

generation/consumption of angular momentum within the system

Empirical Result =====> Angular momentum is conserved !

Conservation of Angular Momentum (about the origin *O*)

rate form:
$$\frac{d\vec{L}_{o,sys}}{dt} = \sum \vec{M}_{o,ext} + \sum_{in} \dot{m}_i (\vec{r} \times \vec{V})_i - \sum_{out} \dot{m}_e (\vec{r} \times \vec{V})_e$$

Angular Impulse

SPECIAL CASE: Plane Motion (Two-dimensional) of a Closed, Rigid System

angular momentum about origin *O*: $\boxed{\vec{L}_{o,sys} = \vec{r}_G \times m\vec{V}_G + I_G\vec{w}}$

where $\vec{r}_G =$ the position vector of the center of mass with respect to the origin.

Conservation of Angular Momentum: $\boxed{\dfrac{d\vec{L}_{o,sys}}{dt} = (\vec{r}_G \times m\vec{V}_G) + I_G\,\vec{a} = \sum \vec{r} \times \vec{F}_{ext}}$

where $\vec{r} =$ the position vector with respect to the origin.

2. Apply conservation of angular momentum to solve problems involving
   (1) steady-state open or closed systems,
   (2) static (stationary) closed system,
   (3) closed, stationary, rigid-body systems,
   (4) translating, closed, rigid body systems, i.e. systems with ω=0 and α=0.

(See item number 2 on the linear momentum objectives page to see necessary steps.)