Name ______________________________

EM406
Examination III
November 7, 2003

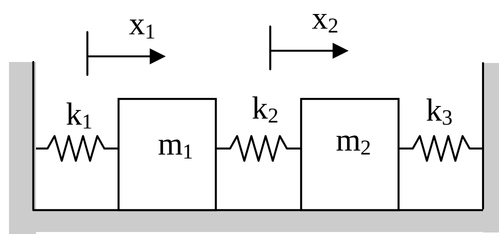
Problem	Score
1	/15
2	/20
3	/40
4	/25
Total	/100

Show all work for credit
AND
Stay in your seat until the end of class
AND
Turn in your signed help sheet

Name ___________________________________ 15 pts

EM406 Examination III **Problem 1** November 7, 2003

The equations of motion for the 2-DOF system shown is



$$\begin{bmatrix} 5 & 0 \\ 0 & 1 \end{bmatrix} \begin{Bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{Bmatrix} + \begin{bmatrix} 60 & -20 \\ -20 & 20 + k_3 \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix} = \begin{Bmatrix} F \sin 10t \\ 0 \end{Bmatrix}$$

Determine an appropriate value of k_3 so that m_1 is stationary, that is, so that m_2 acts as a vibration absorber for m_1.

Name ___________________________________ 20 pts

EM406 Examination III **Problem 2** November 7, 2003

A coworker asks your help in doing a modal analysis problem. The mass matrix and modal matrix that he has determined are shown below. Answer the following questions.

$$[\phi] = \begin{bmatrix} 0.425 & -0.75 & 0.263 \\ 0.526 & 0 & -0.851 \\ 0.425 & 0.75 & 0.263 \end{bmatrix} \text{ and mass matrix } [M] = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

Are these modes mass normalized (note that there will be some round-off error due to how many significant figures I kept)? If not, mass normalize the modes that are not mass normalized.

He has a second problem where he has determined that

$$[K] = \begin{bmatrix} 4 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix} \text{and the modal matrix is} [\phi] = \begin{bmatrix} 1 & -1 & 1 \\ 3 & 0 & -1 \\ 2 & 1 & \textit{aargh} \end{bmatrix}$$

As he was writing down the modal matrix his computer crashed right before he had finished. Determine the last element of the third mode.

Name ___________________________________ 40 pts

EM406 Examination III **Problem 3** November 7, 2003

A three degree of freedom system is governed by the equations shown below. Determine

a) the decoupled equations of motion in terms of normal coordinates, $q_i(t)$ (write out the decoupled equations with numbers substituted in)

b) the initial conditions for the normal coordinates.

$$\begin{bmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix} \begin{Bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \\ \ddot{x}_3 \end{Bmatrix} + \begin{bmatrix} 3 & -2 & 0 \\ -2 & 4 & -2 \\ 0 & -2 & 3 \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \end{Bmatrix} = \begin{Bmatrix} 3\cos 4t \\ 0 \\ 0 \end{Bmatrix}$$

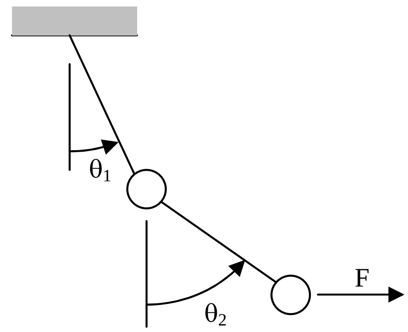
$$x_1(0) = x_2(0) = x_3(0) = \dot{x}_1(0) = \dot{x}_3(0) = 0$$

$$\dot{x}_2(0) = 2$$

Name ___________________________________ 25 pts

EM406 Examination III **Problem 4** November 7, 2003

For the double pendulum shown, the kinetic energy, potential energy and virtual work are:



$$T = \frac{1}{2} m_1 \ell_1^2 \dot{\theta}_1^2 + \frac{1}{2} m_2 \left(\ell_1^2 \dot{\theta}_1^2 + 2\ell_1 \ell_2 \dot{\theta}_1 \dot{\theta}_2 \cos(\theta_1 - \theta_2) + \ell_2^2 \dot{\theta}_2^2 \right)$$

$$V = -m_1 g \ell_1 \cos\theta_1 - m_2 g(\ell_1 \cos\theta_1 + \ell_2 \cos\theta_2)$$

$$\delta W = F\ell_1 \cos\theta_1 \delta\theta_1 + F\ell_2 \cos\theta_2 \delta\theta_2$$

Using Lagrange's equations determine <u>the equation of motion for θ_2</u>.