

Name _____

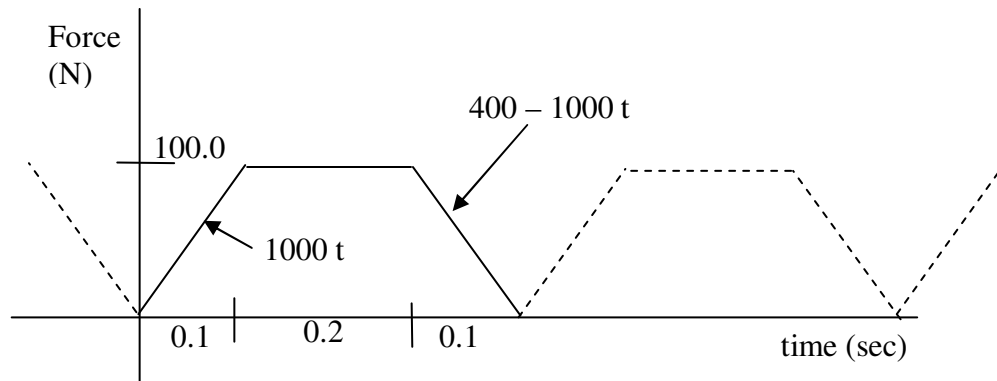
EM406
Examination II
October 25, 2007

Problem	Score
1	/30
2	/40
3	/30
Total	/100

Show all work for credit
AND
Stay in your seat until the end of class
AND
Turn in your signed help sheet

NOTE: Don't get bogged down on short answer problems! You should only spend about 15 minutes on these problems!

Problem 1.1 A lightly damped SDOF linear dynamic system has a natural frequency of 20.0 Hz. It is loaded by the periodic forcing function shown below. **You should not need your Maple worksheet for any of these problems.**



a) The peak amplitude of the periodic function shown above is 100.0 N. Calculate the Fourier coefficient, a_0 . (Hint: Maple is NOT needed.) (3 pts)

b) When you apply this forcing to a system you get the following results from Maple for the output. What is the problem with these results? (2 pts)

$$\begin{aligned}
 &0.00118 - 0.000642 \cos(15.7 t - 0.0000225) + 0.00000103 \sin(15.7 t - 0.0000225) - 0.000329 \cos(31.4 t - 0.0000455) \\
 &+ 0.00000104 \sin(31.4 t - 0.0000455) - 0.0000754 \cos(47.1 t - 0.0000696) + 3.56 \cdot 10^{-7} \sin(47.1 t - 0.0000696) \\
 &+ 3.37 \cdot 10^{-7} \cos(62.8 t - 0.0000954) - 0.00000259 \sin(62.8 t - 0.0000954) - 0.0000263 \cos(78.5 t - 0.000124) \\
 &+ 2.81 \cdot 10^{-7} \sin(78.5 t - 0.000124)
 \end{aligned}$$

c) Write the Maple specification of the function shown above. (2 pts)

d) Estimate approximately how many terms you need to include in the Fourier series to model the steady state response? DO NOT use a Maple worksheet to answer this problem. (5 pts)

Problem 1.2 (3 pts)

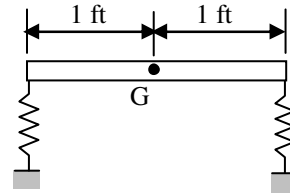
Given a set of FRFs for a structure how can you determine a good approximation to the mode shape?

Problem 1.3 (5 pts) The system shown below is described by x_G which is the displacement of the center of gravity measured positive up, and θ , which is the angle of the bar measured positive in a counter-clockwise direction. You find the equations of motion are:

$$[M] \begin{Bmatrix} \ddot{x}_G \\ \ddot{\theta} \end{Bmatrix} + [K] \begin{Bmatrix} x_G \\ \theta \end{Bmatrix} = 0$$

When Matlab is used to obtain the natural modes you find

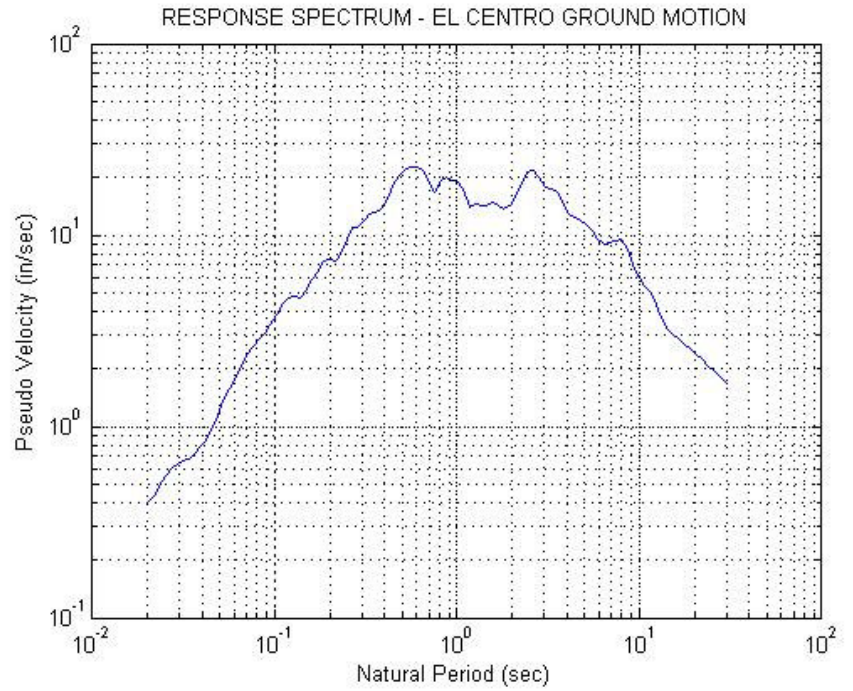
$$\text{Mode 1} = \begin{Bmatrix} 0.15 \\ 0.05 \end{Bmatrix}$$



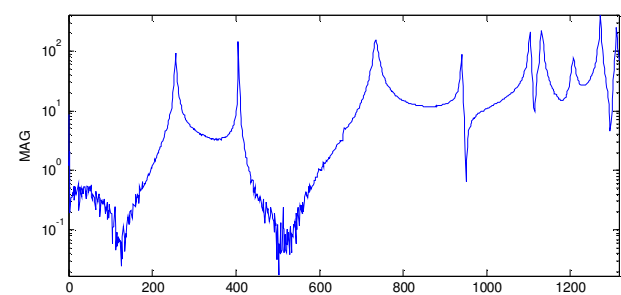
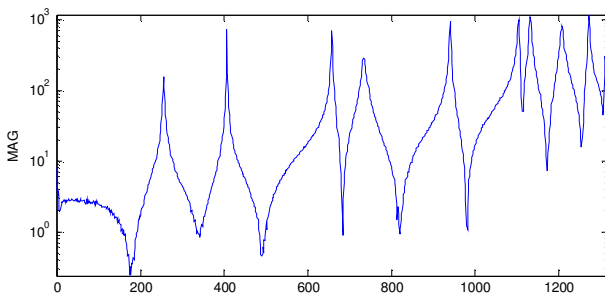
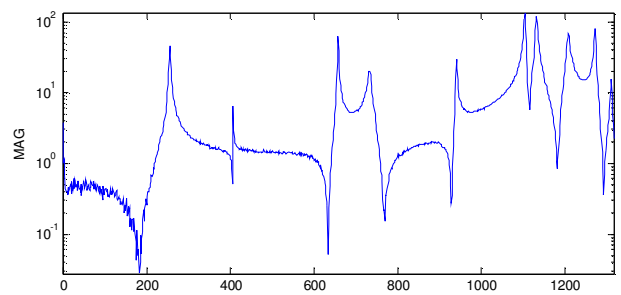
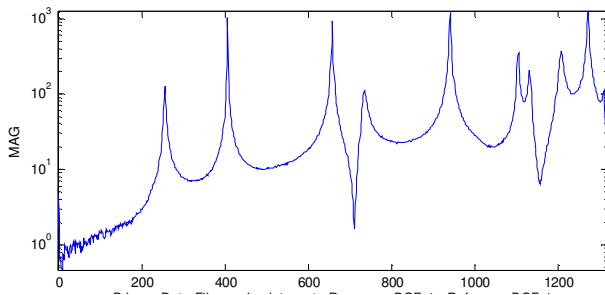
Accurately sketch this system in the first mode shape. Be sure to label the relative displacements of each end of the beam clearly.

Problem 1.4 What is a response spectrum? (2 pts)

Problem 1.5 A tower is modeled as a vertical beam having stiffness of 25000 N/m and a tip mass of 1000 kg giving it a natural frequency of about 5 rad/s. Predict the maximum deflection of this structure under a response spectrum load derived from the El Centro Earthquake. Show your work on the enclosed figure. (4 pts)



Problem 1.6 Which of the four FRFs below corresponds to a drive point measurement? (2 pts)
Circle the correct one. How do you know? (2 pts)



You find that a system has the equations of motion:

$$\begin{bmatrix} 2 & 0 \\ 0 & 0.5 \end{bmatrix} \begin{Bmatrix} \ddot{\theta} \\ \ddot{x} \end{Bmatrix} + \begin{bmatrix} 1000 & -100 \\ -100 & 410 \end{bmatrix} \begin{Bmatrix} \theta \\ x \end{Bmatrix} = \begin{Bmatrix} 0 \\ f(t) \end{Bmatrix}$$

Determine:

- a) The natural frequencies and modes for this system. You may use Maple or Matlab, but show enough work so I know you could do it by hand if necessary. I want a numerical answer for this part of the problem.

For the next two parts I DO NOT WANT NUMERICAL ANSWER. For each part, all I want is a set of scalar equations and a list of unknowns.

- b) Determine the steady state response assuming $f(t) = 8 \cos 20t$
c) Determine the homogeneous solution assuming $f(t) = 0$ and the initial conditions:
 $x(0) = 1, \dot{x}(0) = 2, \theta(0) = 3, \dot{\theta}(0) = 4$

The 2-DOF system shown below is forced harmonically with the force $f(t) = F \sin \omega t$.

- a) Using Lagrange's equations determine the equations of motion for the two masses shown using the coordinates shown and assuming small angles.

