

Homework – Lecture2

Problem 2.1

The force-displacement curve for a non-linear spring was measured experimentally and the data is shown below in Table 1. The displacement was measured from the undeflected length of the spring.

Table 1 – Data for a non-linear spring	
Displacement (in)	Force (lbs)
0.0	0.0
0.1	0.69
0.2	1.52
0.3	2.60
0.4	4.09
0.5	6.10
0.6	8.77
0.7	12.24
0.8	16.62
0.9	22.07
1.0	28.70

- Plot the force-displacement data
- Find the linear equation of motion for small oscillations about the static equilibrium point for the spring mass systems shown in Figure 1. The mass weights 10 lbs. Write the equation of motion in terms of the coordinate z where z is the displacement from the static equilibrium position. What is the natural frequency for small vibrations about the static equilibrium point?
- If the spring mass system is placed on its side as shown in Figure 2, what is the natural frequency for small oscillations about the static equilibrium point?

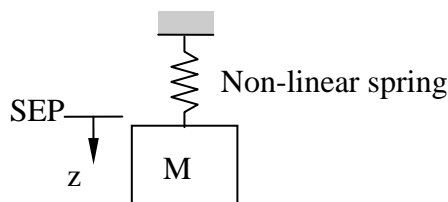


Figure 1

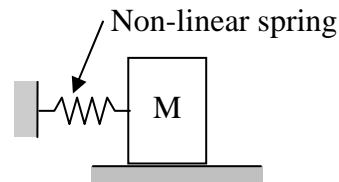
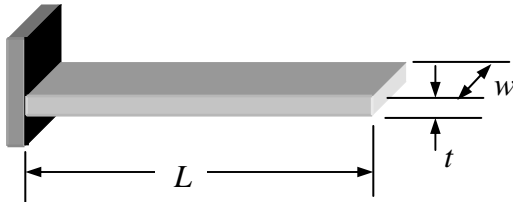


Figure 2

Problem 2.2

To enhance the power harvesting capabilities of a piezoelectric (PZT) patch you have been asked to design an auxiliary structure tuned to the first natural frequency of a frame, f_1 (Hz). The PZT patch will be attached to the auxiliary structure. Assume the auxiliary structure is a cantilevered beam and is to be made of a material with Young's modulus, E , density, ρ , thickness, t , and width, w as shown below.



- Determine a design equation for the length of the beam, L , in terms of the other system parameters (E , ρ , t , w) and f_1 assuming the cantilevered beam can be modeled as a single degree-of-freedom system using the equations given in the modeling handout. The area moment of inertia of a rectangular cross section is $I = wt^3/12$.
- If the first natural frequency of the frame is 14 Hz and you have an aluminum sheet that has a width of 2" and a thickness of 1/16" determine the required length of the beam.

