

Fourier Series Example

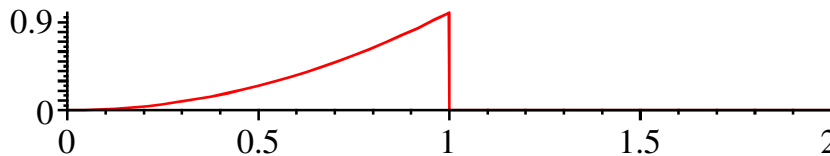
$$T := 2; \tag{1}$$

$$w_0 := 2 \cdot \pi / T; \tag{2}$$

$$g := \text{piecewise}(t < 1, t^2, t < 2, 0); \tag{3}$$

$$\text{piecewise}(t < 1, t^2, t < 2, 0)$$

$$\text{plot}(g, t=0..T);$$



$$a_0 := 1/T \cdot \int_0^T g(t) dt; \tag{4}$$

$$\frac{1}{6}$$

$$a_n := n \rightarrow 2/T \cdot \int_0^T g(t) \cos(n w_0 t) dt; \tag{5}$$

$$n \frac{2 \left(\int_0^T g \cos(n w_0 t) dt \right)}{T}$$

$$b_n := n \rightarrow 2/T \cdot \int_0^T g(t) \sin(n w_0 t) dt; \tag{6}$$

$$n \frac{2 \left(\int_0^T g \sin(n w_0 t) dt \right)}{T}$$

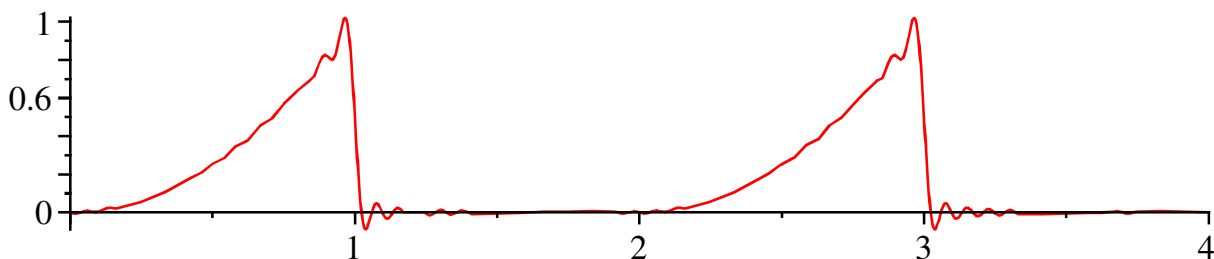
$$f := m \rightarrow a_0 + \sum_{k=1}^m (a_n(k) \cos(k w_0 t) + b_n(k) \sin(k w_0 t)); \tag{7}$$

$$m \quad a_0 \quad \left(a_n(k) \cos(k w_0 t) + b_n(k) \sin(k w_0 t) \right)$$

$$\text{evalf}(f(6), 3); \tag{8}$$

$$\begin{matrix} 0.167 & 0.202 \cos(3.14 t) & 0.189 \sin(3.14 t) & 0.0505 \cos(6.28 t) & 0.159 \sin(6.28 t) \\ 0.0224 \cos(9.42 t) & 0.101 \sin(9.42 t) & 0.0126 \cos(12.6 t) & 0.0795 \sin(12.6 t) \\ 0.00808 \cos(15.7 t) & 0.0625 \sin(15.7 t) & 0.00562 \cos(18.8 t) & 0.0531 \sin(18.8 t) \end{matrix}$$

$$\text{plot}(f(26), t=0..2 \cdot T);$$



Use this as forcing to a physical system

TF:=1/(s^2+2*s+100);

$$\frac{1}{s^2 + 2s + 100} \quad (9)$$

Find the magnitude and phase after substituting in I*omega

mag:=omega->abs(subs(s=I*omega,TF));
 $|subs(s=I,TF)|$ (10)

angletf:=omega->argument(subs(s=I*omega,TF));
 $argument(subs(s=I,TF))$ (11)

Now find the time response

y:=j->a0*mag(0)+sum(an(m)*mag(m*w0)*cos(m*w0*t+angletf(m*w0))+
bn(m)*mag(m*w0)*sin(m*w0*t+angletf(m*w0)),m=1..j);

$$j \quad a_0 \text{mag}(0) + \sum_{m=1}^j (a_n(m) \text{mag}(m \omega_0) \cos(m \omega_0 t + \text{angletf}(m \omega_0)) + b_n(m) \text{mag}(m \omega_0) \sin(m \omega_0 t + \text{angletf}(m \omega_0))) \quad (12)$$

Evaluate the first 5 terms to 5 decimal places

evalf(y(5),5);

$$\begin{aligned} &0.0016667 \quad 0.0022428 \cos(3.1416 t \quad 0.069599) \quad 0.0020953 \sin(3.1416 t \quad 0.069599) \\ &0.00081960 \cos(6.2832 t \quad 0.20473) \quad 0.0025748 \sin(6.2832 t \quad 0.20473) \\ &0.0010275 \cos(9.4248 t \quad 1.0357) \quad 0.0046241 \sin(9.4248 t \quad 1.0357) \\ &0.00020059 \cos(12.566 t \quad 0.40940) \quad 0.0012604 \sin(12.566 t \quad 0.40940) \\ &0.000054013 \cos(15.708 t \quad 0.21091) \quad 0.00041734 \sin(15.708 t \quad 0.21091) \end{aligned} \quad (13)$$

Plot the result using the first 6 terms

plot(y(6),t=0..2*T);

