Fourier Series Example

Define the function

The only thing in this section that needs to be changes is the period, T, and the definition of the function, g.

restart

$T \quad 2$

$$T := 2 \qquad \textbf{(1.1.1)}$$

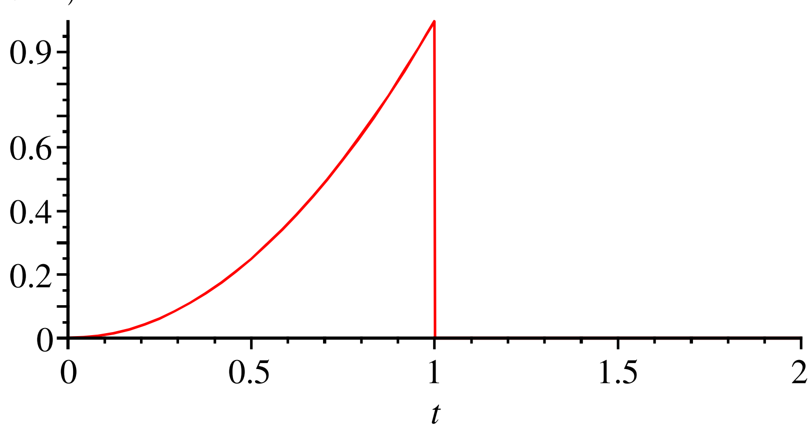
$w0 := \dfrac{2}{T}$

$$w0 := \qquad \textbf{(1.1.2)}$$

$g := piecewise\left(t \quad 1, t^2, t \quad 2, 0\right)$

$$g := \begin{cases} t^2 & t \quad 1 \\ 0 & t \quad 2 \end{cases} \qquad \textbf{(1.1.3)}$$

$plot(g, t = 0..T)$



Define the Fourier Coefficients and the Fourier Series

None of the lines below need to be changed.

$a_0 \qquad \dfrac{1\left(\int_0^T g\,dt\right)}{T}$

$$a_0 := \frac{1}{6} \qquad \textbf{(1.2.1)}$$

$$a_n \quad n \quad \frac{2\left(\int_0^T g\cos(n\, w0\, t)\, \mathrm{d}t\right)}{T}$$

$$a_n := n \quad \frac{2\left(\int_0^T g\cos(n\, w0\, t)\, \mathrm{d}t\right)}{T} \qquad \textbf{(1.2.2)}$$

$$b_n \quad n \quad \frac{2\left(\int_0^T g\sin(n\, w0\, t)\, \mathrm{d}t\right)}{T}$$

$$b_n := n \quad \frac{2\left(\int_0^T g\sin(n\, w0\, t)\, \mathrm{d}t\right)}{T} \qquad \textbf{(1.2.3)}$$

Now let's calculate the Fourier Series using these coefficients

$$f := m \quad a_0 \quad \sum_{k=1}^{m} \left(a_n(k)\cos(k\, w0\, t) \quad b_n(k)\sin(k\, w0\, t) \right)$$

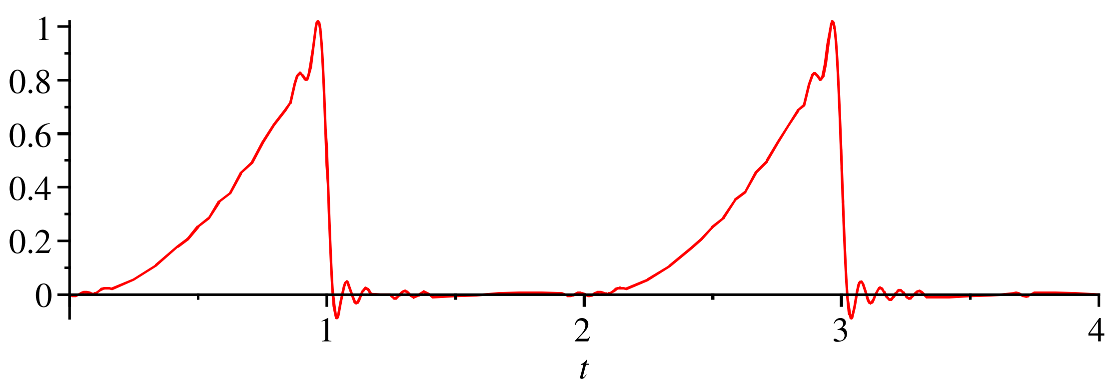
$$f := m \quad a_0 \quad \sum_{k=1}^{m} \left(a_n(k)\cos(k\, w0\, t) \quad b_n(k)\sin(k\, w0\, t) \right) \qquad \textbf{(1.2.4)}$$

We can now find the first "m" terms of the Fourier series by writing f(m) as shown below. The "evalf(f(6),4) just says to calculate the first 6 terms using 4 digits. Plot(f(26), t = 0.. 2T) is plotting the first 26 terms. These can be changed to look at as many terms as you'd like.

$evalf(f(6), 4)$

$$0.1667 \quad 0.2026\cos(3.142\, t) \quad 0.1893\sin(3.142\, t) \quad 0.05065\cos(6.284\, t) \quad 0.1592\sin(6.284\, t) \quad 0.02251\cos(9.426\, t) \quad 0.1013\sin(9.426\, t) \quad 0.01266\cos(12.57\, t) \quad 0.07958\sin(12.57\, t) \quad 0.008104\cos(15.71\, t) \quad 0.06262\sin(15.71\, t) \quad 0.005628\cos(18.85\, t) \quad 0.05306\sin(18.85\, t) \qquad \textbf{(1.2.5)}$$

$plot(f(26), t = 0..2\, T)$



Calculate the magnitude and phase of the Fourier Components (if desired)

Use this function as forcing to a physical system

Define the transfer function (this is the only line that needs to be changed in this section)

$$TF := \frac{1}{s^2 \quad 2\,s \quad 100}$$

$$TF := \frac{1}{s^2 \quad 2\,s \quad 100} \qquad \textbf{(1.4.1)}$$

Find the magnitude and phase of the transfer function at s = jw

$$mag := \quad \left| subs(s = I \quad , TF) \right|$$

$$mag := \quad \left| subs(s = \mathrm{I} \quad , TF) \right| \qquad \textbf{(1.4.2)}$$

$$angletf := \quad \mathrm{argument}(subs(s = I \quad , TF))$$

$$angletf := \quad \mathrm{argument}(subs(s = \mathrm{I} \quad , TF)) \qquad \textbf{(1.4.3)}$$

$$y := j \quad a_0\, mag(0) \quad \sum_{m=1}^{j} \big(a_n(m)\, mag(m\, w0) \cos(m\, w0\, t \quad angletf(m\, w0)) \quad b_n(m)\, mag(m\, w0) \sin(m\, w0\, t \quad angletf(m\, w0)) \big)$$

$$y := j \quad a_0\, mag(0) \quad \sum_{m=1}^{j} \big(a_n(m)\, mag(m\, w0) \cos(m\, w0\, t \quad angletf(m\, w0)) \quad b_n(m)\, mag(m\, w0) \sin(m\, w0\, t \quad angletf(m\, w0)) \big) \qquad \textbf{(1.4.4)}$$

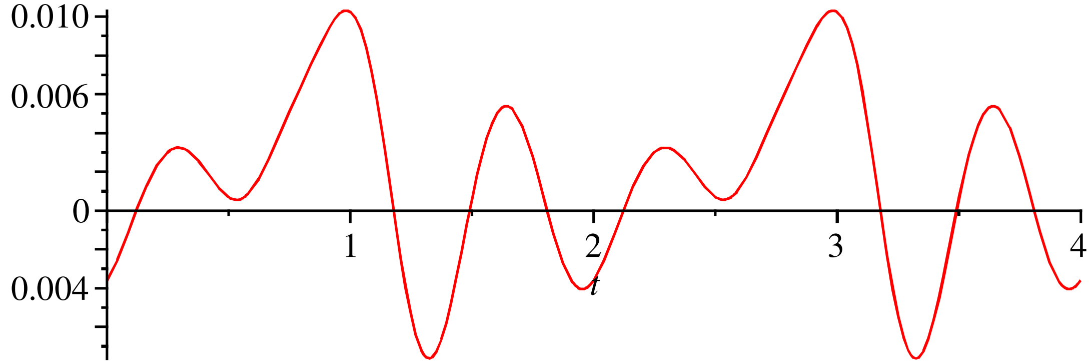
Let's look at the first 6 terms of the steady-state response using 5 decimal places.

$$evalf(y(6), 5)$$

$$0.0016667 \quad 0.0022428 \cos(3.1416\, t \quad 0.069599) \quad 0.0020953 \sin(3.1416\, t \quad 0.069599) \quad 0.00081960 \cos(6.2832\, t \quad 0.20473) \quad 0.0025748 \sin(6.2832\, t \quad 0.20473) \quad 0.0010275 \cos(9.4248\, t \quad 1.0357) \quad 0.0046241 \sin(9.4248\, t \quad 1.0357) \quad 0.00020059 \cos(12.566\, t \quad 0.40940) \quad 0.0012604 \sin(12.566\, t \quad 0.40940) \quad 0.000054013 \cos(15.708\, t \quad 0.21091) \quad 0.00041734 \sin(15.708\, t \quad 0.21091) \quad 0.000021811 \cos(18.850\, t \quad 0.14660) \quad 0.00020557 \sin(18.850\, t \quad 0.14660) \qquad \textbf{(1.4.5)}$$

Let's plot the steady state response. To change the number of terms just change the number in parenthesis after y.

$$plot(y(6), t = 0..2\, T)$$


0.010
0.006
0
0.004
1
2
3
4
t