RHIT / Department of Humanities & Social Sciences / K. Christ
Spring 2011 – 2012 / IA 350, Intermediate Microeconomics / Problem Set 3

1. Suppose a consumer earns income of (m_1, m_2) in periods 1 and 2, and uses this income to finance consumption of (c_1, c_2) in the two periods. Further suppose that this individual may borrow or lend at an interest rate of *r*.

   a. Write down this person's intertemporal budget constraint in present value terms.

   b. If this consumer does not consume anything in period 1, what is the maximum amount he or she can consume in period 2? What do we call this amount?

   c. If this consumer does not consume anything in period 2, what is the maximum amount he or she can consume in period 1? What do we call this amount?

   d. Construct a diagram illustrating this consumer's situation. Indicate the slope of the consumer's budget constraint.

2. Suppose that a consumer has income of $40,000 in period 1 (working years), and income of $11,000 in period 2 (retirement), that he or she may borrow or lend at a prevailing interest rate, *r*, and that there is no inflation.

   a. If this consumer's utility function is $U(c_1, c_2) = c_1 c_2$ and the interest rate is 10%, find the optimal consumption bundle, and state clearly whether this consumer borrows or lends in period 1.
   b. What happens if the interest rate goes up to 15%? Can you tell if the consumer borrows or lends more or less? Calculate his new optimal consumption bundle after this change.
   c. Is this consumer better off or worse off after the rise in the interest rate? How can you tell?
   d. Briefly describe how your solutions in parts (a) and (b) illustrate the permanent income / life cycle model of consumption.

3. The table below reports the inflation rate and the annual rate of return on treasury bills in several countries for the years 1984 and 1985:

Country	Inflation Rate, 1984	Inflation Rate, 1985	Nominal Interest Rate, 1984	Nominal Interest Rate, 1985	Real Interest Rate, 1984	Real Interest Rate, 1985
United States	3.6%	1.9%	9.6%	7.5%		
Israel	304.6%	48.1%	217.3%	210.1%		
Switzerland	3.1%	0.8%	3.6%	4.1%		
W. Germany	2.2%	-0.2%	5.3%	4.2%		
Italy	9.2%	5.8%	15.3%	13.9%		
Argentina	90%	672%	NA	NA	NA	NA
Japan	0..6%	2.0%	NA	NA	NA	NA

   a. Use the textbook formula to compute the exact real interest rates, and enter these values in columns 6 and 7.

b. What would the nominal interest rate in Argentina have to have been to yield a real rate of return of 5% in 1985? _____.

What would the nominal interest rate in Japan have to have been to yield a real rate of return of 5% in 1985? _____.

c. When inflation is low, subtracting the inflation rate from the nominal rate of return gives a good approximation to the real interest rate. For the United States in 1984, this approximation means that the real interest rates was _____ in 1984.

When inflation is very high, subtracting the inflation rate from the nominal rate of return gives a poor approximation to the real interest rate. For Israel in 1984, this approximation implies that the real interest rate was _____ in 1984. Furthermore, for Argentina in 1985, this approximation implies that a nominal interest rate of _____ would have yielded a real interest rate of 5%. (This should be in contrast with the answer you gave in part b).

d. Consider an investment opportunity in the United States in 1985 that guaranteed $1 million to be delivered 10 years in the future. What would have been a reasonable value to place on such an investment opportunity in 1985 in the United States? (Calculate the present value of a future amount.)

e. Consider the same investment opportunity at the same time in Italy. What would have been a reasonable value to place on it in Italy?

4. What will be the amount accumulated by each of these present investments?
   a. $5,635 in 10 years at 5% compounded semiannually.
   b. $7,500 in 15 years at 6% compounded quarterly.
   c. $38,300 in 7 years at 9% compounded monthly.

5. Calculate the present value (in April 2012) of a 15-year Ford bond (Issued April 2003, Maturity date, April 2018), $1,000 face or "par" value, with a 12% coupon rate, and annual interest payments if the relevant discount rate is 5%, 10%, and 15%.

6. Suppose a $1,000 bond (face value) pays a 7% annual coupon rate and matures in 3 years.

   a. How much is the bond worth if a relevant discount rate is 10%? _____.

   b. Suppose that the Federal Reserve <u>unexpectedly</u> announces another cut in interest rates. What do you expect will happen to the market price of this bond after this announcement?

   _____.

7. Chillingsworth owns a large, poorly insulated home. His annual fuel bill for home heating averages $800 per year. An insulation contractor suggests to him the following options:

   - Plan A. Insulate just the attic. If he does this, he will permanently reduce his fuel consumption by 15%. Total cost of insulating the attic is $1,000.
   - Plan B. Insulate the attic and the walls. If he does this, he will permanently reduce his fuel consumption by 20%. Total cost of insulating the attic and the walls is $1,600.
   - Plan C. Insulate the attic and the walls, and install a solar heating unit. If he does this, he will permanently reduce his fuel costs to zero. Total cost of this option is $15,000 for the solar heater and $500 for the insulation.

   a. Assume for simplicity of calculations that the house and the insulation will last forever. Calculate the present value of the dollars saved on fuel from each of the three options if the relevant discount rate is 10%.
   b. Each plan requires an expenditure of money to undertake. Calculate the difference between the present value and the present cost of each plan.
   c. If the price of fuel is expected to remain constant, which option should Chillingsworth choose if he can borrow and lend at an annual interest rate of 10%?
   d. Which option should he choose if he can borrow and lend at an annual rate of 5%?
   e. Suppose that the government offers to pay half the cost of any insulation or solar heating device. Which option would he now choose if his borrowing and lending rate is 10%? Which option would he now choose if his borrowing and lending rate is 10%?
   f. Suppose that there is no government subsidy but that fuel prices are expected to rise by 5% per year. What is the present value of fuel savings from each of the three proposals if the relevant discount rate is 10%? Which proposal should Chillingsworth choose if his borrowing and lending rate is 10%? Which proposal should Chillingsworth choose if his borrowing and lending rate is 10%?

8. An area of land has been planted with Christmas trees. On December 1, ten years from now, the trees will be ready for harvest. At that time, the standing Christmas trees can be sold for $1,000 per acre. The land, after the trees have been removed, will be worth $200 per acre. There are no taxes or operating expenses, but also no revenue from this land until the trees are harvested. The interest rate is 10%.

   a. What can we expect the market price of the land to be? _____

   b. Suppose that the Christmas trees do not have to be sold after 10 years, but could be sold in any year. Their value if they are cut before they are 10 years old is zero. After the trees are 10 years old, an acre of trees is worth $1,000 and its value will increase by $100 per year for the following 10 years. After the trees are cut, the land on which the trees stood can always be sold for $200 per acre. When should the trees be cut to maximize the present value of the payments received for

      trees and land? _____

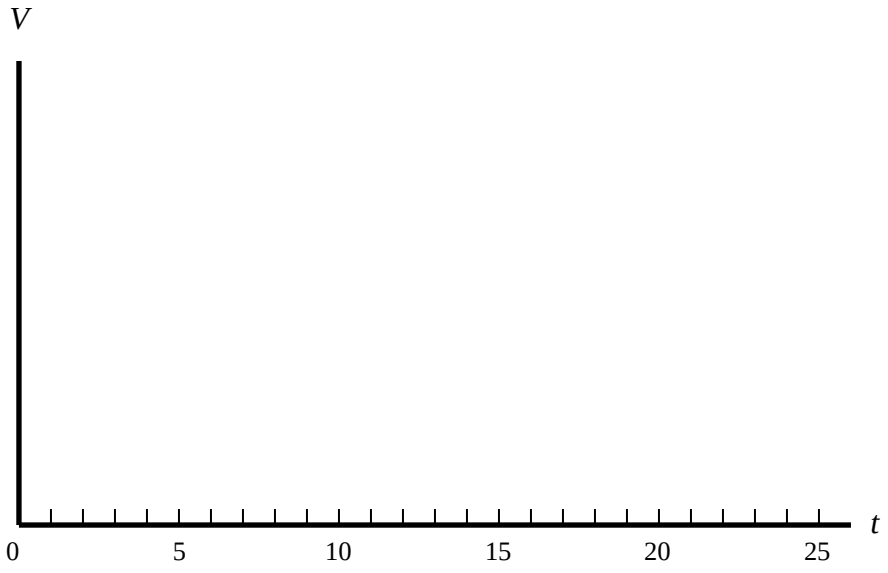
      Under these conditions, what can we expect the market price of the land to be? _____

9. Consider an asset whose market value, *V*, will evolve over time according to $V(t) = 1.5t - 0.05t^2$, where *t* is years.

   a. In how many years will this asset's market value reach its maximum? Modify the diagram below by adding a function that depicts the behavior of this asset over time, labeling your function $V(t)$.

   b. If an investor can borrow and lend in capital markets at a market-determined annual rate of interest of 10%, write out an equation for determining how long the investor should hold the asset described in part (a). You do not have to solve for the optimal holding period – just write out the equation.

   c. Add a second function to the diagram below, labeling it $R(t)$, where this second function approximates the behavior of an initial amount invested at the annual 10% market rate of interest. After adding this second function, indicate *on the horizontal axis* the approximate optimal holding period for the asset described in part (a).



10. Annualizing returns and volatility:

   a. If average daily returns are 0.0004 (0.04%) and the standard deviation of daily returns is 0.01 (1%), calculate the annualized return and annual volatility.

   b. If average weekly returns are 0.003 (0.3%) and the standard deviation of weekly returns is 0.05 (5%), calculate the annualized return and annual volatility.

   c. If average monthly returns are 0.01 (1%) and the standard deviation of monthly returns is 0.06 (6%), calculate the annualized return and annual volatility.

11. Mr. S. Adam owns a small chemical factory, located close to a river that occasionally floods in the spring, with disastrous consequences. Next summer, Mr. Adam plans to sell the factory and retire. The only income he will have is the proceeds of the sale of this factory. If there is no flood, the factory will be worth $500,000. If there is a flood, then what is left of the factory will be worth only $50,000. Mr. Adam can buy flood insurance at a cost of $0.10 for each $1 worth of coverage. Mr. Adam thinks that the probability of a flood this spring is 1 in 10. Let c_F denote the contingent commodity dollars if there is a flood and c_{NF} denote the contingent commodity dollars if there is no flood. Mr. Adam's von Neumann-Morgernstern utility function is $U(c_F, c_{NF}) = \left[pr(c_F) * \sqrt{c_F}\right] + \left[pr(c_{NF}) * \sqrt{c_{NF}}\right]$

    a. To buy insurance that pays him $$x$ in case of a flood, Mr. Adam much pay an insurance premium of 0.1x. If Mr. Adam insures for $$x$, then if there is a flood, he gets $$x$ in insurance benefits. Suppose that Mr. Adam has contracted for insurance that pays him $$x$ in the event of a flood.

    Then after paying his insurance premium, he will be able to consume $c_F =$ _____. If Mr. Adam has this amount of insurance and there is no flood, then he will be able to consume

    $c_{NF} =$ _____.

    b. You may eliminate x from the two equations for c_F and c_{NF} that you wrote out in part (a). This gives you a budget equation for Mr. Adam. The form of the budget equation in which the "price" of c_{NF} is 1 may be written as $0.9c_{NF} +$ _____ $c_F =$ _____

    c. Mr. Adam's marginal rate of substitution between the two contingent commodities, dollars if there is no flood and dollars if there is a flood, is $MRS(c_F, c_{NF}) = -\dfrac{0.1\sqrt{c_{NF}}}{0.9\sqrt{c_F}}$. To find his optimal bundle of contingent commodities, you must set this marginal rate of substitution equal to the number _____. Solving this equation, you find that Mr. Adam will choose to consume the two contingent commodities in the ratio _____.

    d. Since you know the ratio in which he will consume c_F and c_{NF}, and you know his budget equation, you can solve for his optimal consumption bundle, which is _____. In words, Mr. Adam will buy an insurance policy that will pay him _____ if there is a flood. The amount he will have to pay (his insurance premium) will be _____.

12. Donald Rump currently has wealth equivalent to $2,000,000, but believes that he faces a 1% chance of a loss of wealth equal to $500,000.

    a. What is Donald's expected wealth? _____

    b. What is his expected loss? _____

    c. If Donald's utility of wealth may be expressed as $U = \sqrt{WEALTH}$, what is his expected utility when he faces the probable loss? _____

    d. What is Donald's certainty equivalent? _____.

    e. What is his risk premium? _____

    f. If you are an insurance company, what is the highest premium you could charge Sigmund to fully insure him against his loss? _____.

    e. As opposed to part (c), Donald's utility of wealth were U = WEALTH^2, how would you characterize Donald's risk preferences and what would be his risk premium?

    _____

13. Sigmund currently has wealth equivalent to $200,000, but knows that he faces a 20% chance of a loss of wealth equal to $50,000.

    a. What is Sigmund's expected wealth? _____ What is his expected loss? _____

    b. If Sigmund's utility of wealth may be expressed as $U = \sqrt{WEALTH}$, what is his expected utility when he faces the probable loss? _____

    c. What is Sigmund's certainty equivalent? _____. What is his risk premium? _____

    d. If you are an insurance company, what is the highest premium you could charge Sigmund to fully insure him against his loss? _____.

    e. If Sigmund were risk neutral, how would your answer in part (d) change?

14. Suppose that a stock has a beta of 1.5, the return of the market is 10%, and the risk-free rate of return is 5%. What is the expected rate of return this stock according to the CAPM?

15. If the risk-free rate of return is 6%, and if a risky asset is available with an expected return of 9% and a standard deviation of 3%, what is the maximum rate of return you can achieve if you are willing to accept a standard deviation of 2%? What percentage of your wealth would have to be invested in the risky asset?

16. Martha S. has a choice of two assets: The first is a risk-free asset that offers a rate of return of r_f, and the second is a risky asset that has an expected rate of return of r_m and a standard deviation of σ_m.

    a. If x is the percent of wealth that Martha invests in the risky asset, what is the equation for the expected rate of return on the portfolio? What is the equation for the standard deviation of the portfolio?

    _____

    b. By solving the second equation above for x and substituting the result into the first equation, derive an expression for the rate of return on the portfolio in terms of the portfolio's riskiness.

    _____

    c. Suppose that Martha can borrow money at an interest rate of r_f and invest it in the risky asset. If $r_m = 20\%, r_f = 10\%,$ and $\sigma_m = 10\%$, what will be Martha's expected return if she borrows an amount equal to 100% of her initial wealth and invests it in the risky asset?

    _____

    d. Suppose that Martha can borrow or lend at the risk-free rate. If $r_m = 20\%, r_f = 10\%,$ and $\sigma_m = 10\%$, what is the formula for the "budget line" that Martha faces? _____. Plot this budget line below.

    e. Which of the following risky assets would Martha prefer to her present risky asst, assuming she can only invest in one risky asset at a time and that she can invest a fraction of her wealth in whichever risky asset she chooses. Write the word "better", "worse", or "same" after each of these assets:

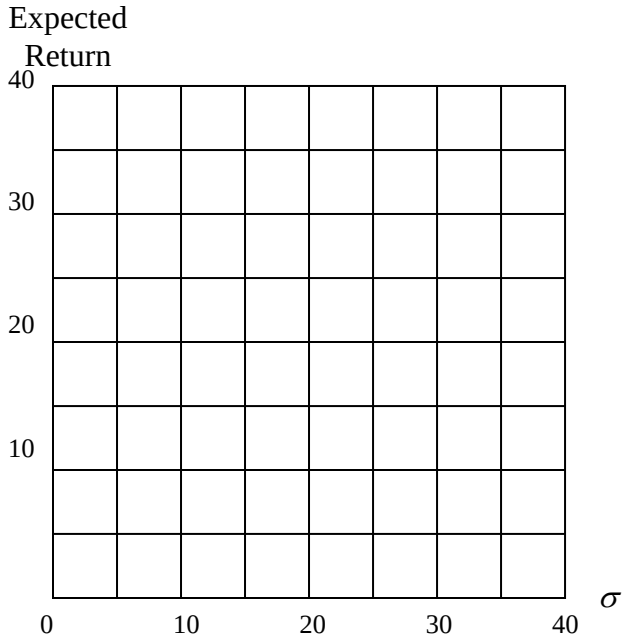
    Asset A, with $r_A = 17\%, , \sigma_A = 5\%$ _____

    Asset B, with $r_B = 30\%, , \sigma_B = 25\%$ _____

    Asset C, with $r_C = 11\%, , \sigma_D = 1\%$ _____

    Asset D, with $r_D = 25\%, , \sigma_D = 14\%$ _____

f. Now suppose that Martha's utility function has the form $U(r_x, \sigma_x) = r_x - 2\sigma_x$. How much of her portfolio will she invest in the original risky asset? _____



17. Assuming that the Capital Asset Pricing Model is valid, complete the following table. In this table, p_0 is the current price of asset *i*, and E[p_1] is the expected price of asset *i* in the next period.

r_f	r_m	r_i	β_i	p_0	E[p_1]
10%	20%	10%		100	
10%	20%		1.5		125
10%		20%	2.0	200	
0%	30%		0.67		48
10%	22%		0	80	

18. Late in the day at an antique rug auction, there are only two bidders left, Arnie and Barney. The last rug is brought out and each bidder examines it. The seller announces that she will sell the rug via a sealed bid, first price action. Both Arnie and Barney believe that the other is equally likely to value the rug at any amount between \$0 and \$1,000. Therefore, for any number X between 0 and 1,000, each of them believes that the probability that the other bidder values the rug at less than X is X/1,000. Arnie actually values the rug at \$800. If he gets the rug, his profit (consumer's surplus) is the difference between \$800 and what he pays for it. He wants to make his bid in such a way as to maximize his expected profit (or consumer's surplus).

    a. Suppose that Arnie thinks that Barney will bid exactly what the rug is worth to him. If Arnie bids \$700, what is the probability that he will get the rug? _____ If he gets the rug for \$700, what is his profit? _____ What is his expected profit from a bid of \$700? _____

    b. If Arnie bids \$600, what is the probability that he will get the rug? _____ If he gets the rug for \$600, what is his profit? _____ What is his expected profit from a bid of \$600? _____

    c. If Arnie bids \$$x$, what is the probability that he will get the rug? _____ If he gets the rug for \$$x$, what is his profit? _____ What is his expected profit from a bid of \$$x$? _____

    d. Given Arnie's private valuation of \$800, what bid x will maximize his expected profit? _____

    e. Let's generalize the result from part (d). If Arnie's valuation is V (and he believes that Barney will bid exactly what the rug is worth to him), write out a formula that expresses Arnie's expected profit in terms of the variables V and x (where x is his optimal bid).
    _____

    Write out a formula for x as a function of V under these conditions.

    _____

    f. Let's generalize a bit further. Your results in parts (a) through (e) should indicate that, under the described conditions, a bidder's optimal strategy is to bid less than his or her private valuation. Describe what you think will happen to the optimal bid, x, as the number of bidders, n, increases. (In the initial part of this problem, $n = 2$, and here is a hint, the general formula for the optimal bid is <u>not</u> $x = \frac{V}{n}$.)

    _____

19. After a delightful quarter studying microeconomics, Milton Keynes, a student at the MIT of west central Indiana, has decided to auction off his used copy of Varian's *Intermediate Microeconomics*. He believes that two kinds of people are interested in buying this book. The first are other students who will be taking Microeconomics next year and who Milton knows would be willing to pay $25 for his book. The second are members of an odd sect of on-line gamers who believe that hidden within the Varian text are secret algorithms that will enable them to achieve mastery of their favorite online game. Milton believes they would be willing to pay $80 for the book. If Milton cannot sell the book at auction, he can always sell it to the used textbook guy for $10.

    Milton has spammed the campus with an "Economics textbook sale today" message, but curiously only two potential buyers have shown up. Milton believes that these two potential buyers are unfamiliar with each other, that they know the distribution of valuations across bidder types, and that the probability that either is a member of the gaming sect is independent of the other's type. He further believes that there is a 50% probability of either of them being a member of this sect.

    Milton considers the following ways of selling the textbook:

    (i) Post a price of $80, and if nobody buys the book at that price, sell it to the used textbook guy.

    (ii) Run an ascending bid English auction with minimum bidding increments of $1 and no reservation (minimum selling) price.

    (iii) Run an ascending bid English auction with minimum bidding increments of $1 and a reservation (minimum selling) price of $80.

    (iv) Run a sealed-bid auction and sell the book to the high bidder at the second highest bid. (If there is a tie, choose one of the bidders at random, and sell the book to this bidder at the price bid by both bidders.)

    a. If Milton sells by method (i), what is his expected revenue? _____

    b. If Milton sells by method (ii), what is his expected revenue? _____

    c. If Milton sells by method (iii), what is his expected revenue? _____

    d. If Milton sells by method (iv), what is his expected revenue? _____

20. In his will, an eccentric uncle has left you a first edition of Adam Smith's *Wealth of Nations*. Since you are a Rose-Hulman student, you have no desire to keep or read this boring old tome, but you do understand that there are odd people in the world who collect books like this and would pay good money for it, so you have decided to sell it at auction in order to help pay off your student loans. Here is what you believe about potential bidders: there are three different types of collectors interested in the book, and their private valuations fall into one of three categories. Casual collectors (type L), who value the book at $20,000; avid collectors (type M), who value the book at $60,000; or braggarts (type H) who are willing to pay $100,000 for the book just so they can show it off at cocktail parties. They are all acquainted with auction theory, and thus understand that their optimal bid in a first-price sealed bid auction would be $b^* = v_i - (v_i - v_l)/n$, where n is the number of bidders who actually participate in the auction.

    On auction day, two bidders show up and you must now decide what type of auction to run. You believe that these two potential bidders are unfamiliar with each other, that they know the distribution of valuations across bidder types, and that the probability that either is a type H, M, or L collector is 1/3, 1/3 , 1/3 respectively. Calculate your expected revenue from running a first price, sealed bid auction and an English auction with $1,000 bid increments:

    a. First price, sealed bid.

    b. English, ascending bid with $1,000 bid increments.

    c. Now suppose that you have no moral scruples, and have three associates that you can call in at the last minute to submit fake bids in the first price, sealed bid auction so that each of the two real bidders think there are five bidders. How much, in total, should you be willing to pay your associates to collude with you in this way?