2.2: Elasticity

To prepare for this lecture, read Hirschey, chapter 4, pages 134 – end, and chapter 11, pages 413 – 417. The material in the first half of Hirschey's chapter 4 is a "rough cut" of consumer demand theory, which is covered in far greater detail in IA350, Intermediate Microeconomics. It will not be covered here, nor are students responsible for it. The material from chapter 11 reviews the concepts of consumer and producer surplus, and dead weight losses.

Relevant types of Elasticity for Managerial Economics
Recalling that any elasticity is merely the ratio of percentage changes in two related variables, we may conceive of many relevant elasticity measures. In Hirschey's extended example in chapter 3, for example, we can think of an "interest rate elasticity of auto demand." Generally, however, in this lecture we consider applications involving three common elasticities:

1. ***Price Elasticity of Demand***: %Δ Quantity Demanded for X / %Δ Price of X
2. ***Cross Price Elasticity of Demand***: %Δ Quantity Demanded for X/ %Δ Price of Y
   For substitute goods, this is positive; for complementary goods, this is negative.
3. ***Income Elasticity of Demand***: %Δ Quantity Demanded for X / %Δ Income
   For inferior goods, this is negative; for normal goods, this is positive; for luxury goods, this is greater than 1. Hirschey division of normal goods is slightly different. Instead of dividing those goods for which income elasticity of demand is positive into normal and luxury goods, he divides them as noncyclical and cyclical normal goods.

<u>Price elasticity of demand, degree of market power, and optimal pricing</u>
In calculating values for any elasticity, we need to distinguish between elasticity measured at a specific point (***point elasticity***) or measured over some range of values (***arc or midpoint elasticity***). To use price elasticity of demand as an example,

$$\epsilon = \frac{\%\Delta Q}{\%\Delta P} = \frac{\Delta Q}{\Delta P} * \frac{P}{Q}$$

Given two price-output combinations, we could calculate three different estimates of price elasticity of demand: at the initial price-quantity combination, at the second price-quantity combination, or over the range of prices and quantities. The first two of these would be point estimates, while the third would be an arc or midpoint estimate. All three would be different. The two point estimates would be calculated as:

$$\epsilon = \frac{\Delta Q}{\Delta P} * \frac{P_1}{Q_1} \quad \text{or} \quad \epsilon = \frac{\Delta Q}{\Delta P} * \frac{P_2}{Q_2}$$

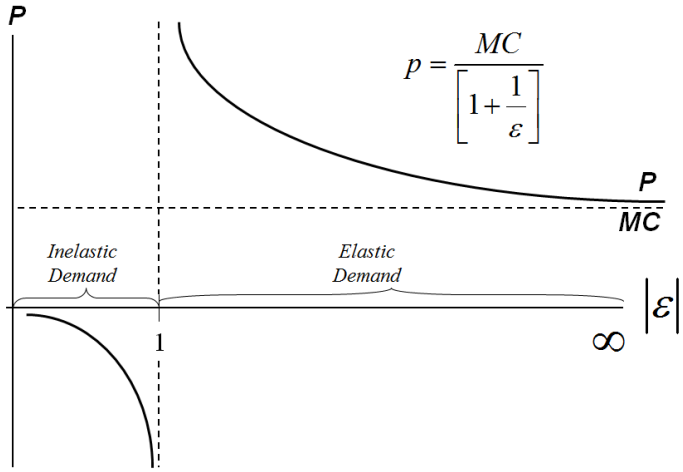
The arc or midpoint estimate would be calculated as:

$$\epsilon = \frac{\Delta Q}{\Delta P} * \frac{P_1 + P_2}{Q_1 + Q_2}$$

In practice, we can make point or arc estimates of any elasticity.

While price elasticity of demand varies along a linear demand curve, we can also model elasticity as being constant along a demand curve. Note that for the nonlinear demand curve $Q = aP^{-b}$, price elasticity of demand is a constant $-b$.

Estimates of price elasticity of demand, coupled with knowledge of marginal costs, can provide firms with the tools necessary for setting optimal prices. The relationship between ***price-cost margins*** and elasticity is summarized by the figure below:



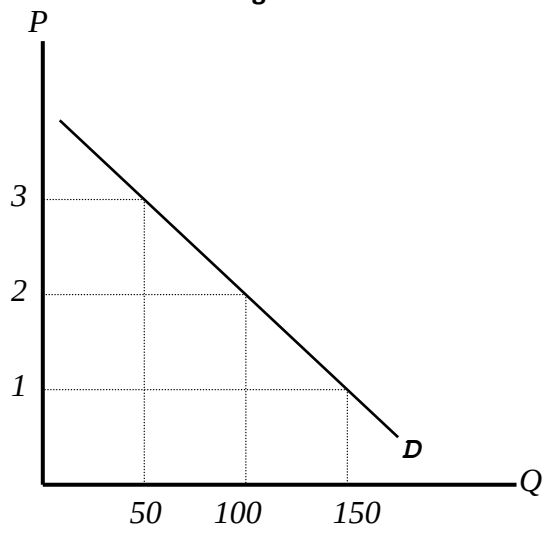
<u>Cross-Price elasticity and multi-product pricing</u>
Estimates of cross-price elasticties for related products in a line of products can provide firms with the tools they necessary for estimating total revenue effects stemming from the change in one price of a family of related goods. The relevant formula for calculating the change in total revenue attributable to the change in one price, when price elasticity of demand and cross-price elasticity of demand is known, is

$$\Delta R = \%\Delta P_x\left[R_x^0(1+\varepsilon_x) + R_y^0(\varepsilon_{x,y})\right]$$

Comparative Statics and Welfare Analysis Using Consumer and Producer Surplus

<u>Consumer surplus</u>
Any "normal" demand curve -- normal in the sense that it is downward sloping -- summarizes the willingness of potential buyers in a market to engage in trades at different prices, and predicts a quantity that will be demanded at any given price. For example, the demand curve in figure 1 predicts that if the price of the good is 2, 100 units will be demanded. Yet the demand curve also informs us that at a price of 2, there are necessarily some potential buyers who are willing to pay more than that. For example, there are 50 potential buyers who are willing to pay at least 3.

Figure 1



Suppose that you are one of these 50 potential buyers. In fact, suppose that, because of your own tastes or preferences, you value the benefits of having one unit of the good at precisely 3. If you are lucky enough to be in the market for the good at a time when the market places a price of 2 for it, then when you buy the good for 2, you experience a gain in your own personal welfare of exactly 1 -- the difference between your "willingness to pay" for one unit and the price of one unit.

This difference in one's willingness to pay for a good and the market-determined price they actually pay is consumer surplus.[1] The simple demand curve in figure 1 summarizes a hypothetical market in which there are many buyers, each of whom may value the good differently. If the market-determined price were 2, then 100 units of the good would be sold (or 100 transactions would take place). The sum of all consumer surplus may be estimated as the area under the demand curve, above the equilibrium price, and up to the equilibrium quantity. This is a triangle, and its area (assuming complete linearity of the demand curve and a market-determined price of 2) is 100.

Thus, we would say that at an equilibrium price of 2, consumers experience welfare gains from their purchases in the amount of 100. Notice that this is not the amount they expend. That is 200 (100 units of the good at a price of 2). Consumer surplus represents the benefits they feel they receive above and beyond the value they give up to obtain the goods.

<u>Producer Surplus</u>
Similarly, any "normal" supply curve -- normal in the sense that it is upward sloping -- summarizes the willingness of potential sellers in a market to engage in trades at different prices, and predicts a quantity that will be supplied at any given price. For example, the supply

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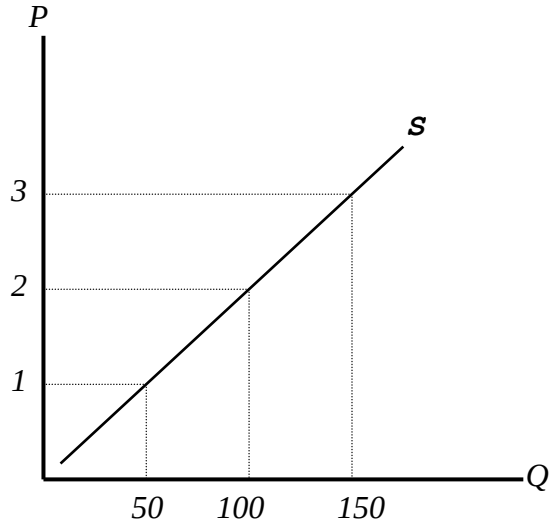
[1] The concept of consumer surplus was first developed by a French engineer, Jules Dupuit, who argued as early as 1844 that consumer surplus could be used to place a value on the benefits of public projects. His logic was simple -- if society pays 500 million for a particular road, then society values the road at a *minimum* of 500 million, and perhaps much more. For details, see an English translation of Dupuit's article "On the measurement of the utility of public works," in Kenneth Arrow and Tibor Skitovsky, eds., *Readings in Welfare Economics*, 1969.

curve in figure 1 predicts that if the price of the good is 2, 100 units will be supplied. Yet the supply curve also informs us that at a price of 2, there are necessarily some potential sellers who are willing to supply some goods to the market at prices below 2. For example, sellers are willing to supply 50 units of the good at a price of 1.

Figure 2



Suppose that you are one of those sellers. In fact, suppose that, because of your own costs of production, you are willing to supply some units of the good to the market if the price is just 1. If the market places a price of 2 on it, then when you sell the good for 2, you experience a gain in your own personal welfare of exactly 1 -- the difference between your "willingness to sell" and the price of one unit.

This difference in a seller's willingness to sell a good and the market-determined price they actually receive is producer surplus. The sum of all producer surplus may be estimated as the area above the supply curve, below the equilibrium price, and up to the equilibrium quantity. This is a triangle, and its area (assuming complete linearity of the supply curve and a market-determined price of 2) is 100. Thus, we would say that at an equilibrium price of 2, producers experience welfare gains from their sales in the amount of 100.

Total Surplus and Economic Welfare

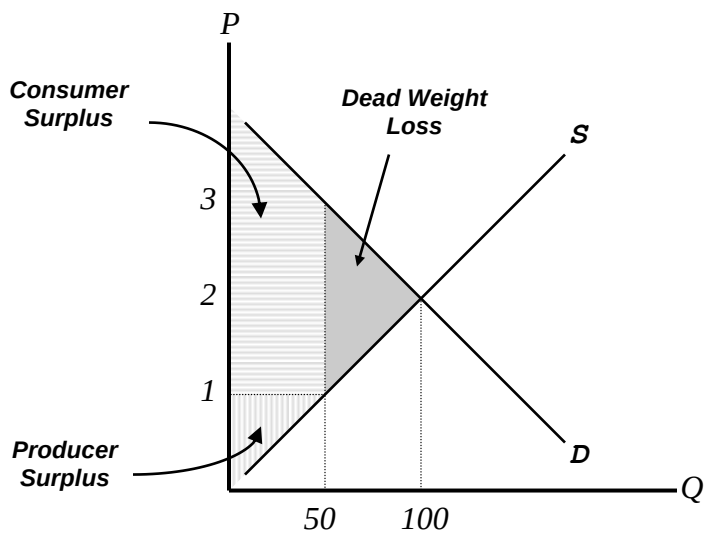
The sum of consumer surplus and producer surplus generated by transactions in a market equals the total surplus in the market. Total surplus is the basic concept that economists use to study the welfare of buyers and sellers in a market. Economists often use these concepts to make normative statements about the desirability of market outcomes. The most important insight in this respect is that unhindered markets lead to outcomes that maximize total surplus.

Consider first that for any quantity less than equilibrium, the value to buyers is greater than the cost to sellers. Thus, expansion of output increases total surplus. This is true for any quantity up to equilibrium. Next, consider that for any quantity greater than equilibrium, the value to buyers is less than the cost to sellers. Thus, expansion of output beyond equilibrium decreases total surplus. This method of analysis leads to the conclusion that free markets maximize total surplus.

The most typical application of surplus to policy analysis is its use in supporting normative statements about market interventions such as the establishment of price ceilings or floors, the imposition of production quotas, or the levying of unit taxes on goods. In most such analysis, it can be shown that a market intervention leads to an outcome with less total surplus than that which would have been generated by a free market. This shortfall in surplus from the idealized free market amount is usually called the "dead weight loss" of the market intervention.

Figure 3 illustrates the creation of a dead weight loss via the imposition of a price ceiling. In the figure, the imposition of a price ceiling at 1 reduces total market transactions from 100 to 50. While this price ceiling increases consumer surplus, it reduces producer surplus and total surplus. One interpretation of this outcome is that while some consumers are better off (they can purchase the good for 1 instead of the free market equilibrium price of 2), there are other consumers who must search for substitutes. This outcome also raises the issue of what new allocation mechanism will develop in lieu of market prices to determine which consumers get the good at the lower administered price.

Figure 3



Relevant Textbook Problems: 4.6, 4.7, 4.8, 4.9, 4.10, 11.3, 11.4