1.5: Standard Production Theory

To prepare for this lecture, read Hirschey, chapter 7.

After an introduction to the general theory of production, this lecture addresses three topics:

2. Factor substitution – Elasticity of Substitution
3. Short-run production theory and diminishing marginal returns

Introduction

Behavior of costs is determined, in part, by the nature of production, which we model as a production function. (The other major factor influencing costs is buyer power in input markets. For most of what follows, we assume that inputs are purchased in competitive markets – there is no buyer power.)

Given a continuous production function, \( q = f(x_1, x_2) \) and a specified level of output \( q^0 \), the production function becomes \( q^0 = f(x_1, x_2) \) and the locus of all combinations of \( x_1 \) and \( x_2 \) form an isoquant. The negative of the slope of an isoquant is defined as the Marginal Rate of Technical Substitution (MRTS).

\[
MRTS = - \frac{dx_2}{dx_1}
\]

The total differential of \( q = f(x_1, x_2) \) is \( dq = f_1 dx_1 + f_2 dx_2 \). Along a given isoquant, \( dq = 0 \). Therefore

\[
f_1 dx_1 + f_2 dx_2 = 0
\]

\[
f_1 dx_1 = - f_2 dx_2
\]

\[
f_1/f_2 = - \frac{dx_2}{dx_1}
\]

and

\[
MRTS = \frac{f_1}{f_2}
\]  

(Hirschey, p. 256)

Conceptually, given a specified level of output \( q^0 \), the firm’s profit maximizing problem becomes one of minimizing costs subject to the constraint imposed by prices of inputs and the isoquant. Visually, think of the firm “pushing down” its budget line until it reaches a point of tangency with the given isoquant.

Cost Minimization and Optimal Input Usage

Let’s say you’ve done your homework and have a fairly good grasp of demand for your product (we’ll address this more fully in lectures 2.1 to 2.4). So you’ve decided upon some amount of that product, \( q \), to produce. Call this specific, pre-determined amount of output \( q_0 \). Furthermore, let’s say that you produce this amount of \( q_0 \) by purchasing a variety of inputs and transforming them into units of \( q \) according to the production function, \( q = f(x_1, x_2, \cdots x_n) \). Finally, let’s say that these inputs have prices that you take as “exogenous” – you don’t get to set them – and your decision to purchase some or even a lot of any of them does not affect their prices. Let these prices be \( [v_1, v_2, \cdots v_n] \). Thus, you can think of your cost function as being the sum of all the inputs you use times their prices (or “costs” to you). Naturally, you would want to produce \( q_0 \) at lowest cost.

We can formalize all this in a mathematical statement, and from that statement make some important observations about what it means to operate efficiently:

\[
\min_{x_1, x_2, \cdots x_n} c(q) \quad \text{where} \quad c(q) = \sum_{i=1}^{n} v_i x_i \quad \text{subject to} \quad q = f(x_1, x_2, \cdots x_n)
\]

The first order conditions for minimization are ...
v_1 - \lambda f_1 = 0, v_2 - \lambda f_2 = 0, \ldots, v_1 - \lambda f_1 = 0, \text{ and } q_0 - f(x_1, x_2, \ldots, x_n) = 0

where f_i = MP_i, and from which we can easily observe a condition necessary for optimal input usage:

\[
\frac{MP_1}{v_1} = \frac{MP_2}{v_2} = \cdots = \frac{MP_n}{v_n}
\]

Notice that this same rule was derived in Lecture 1.2 from a profit maximization formalization.

**Elasticity of (Factor) Substitution**

The elasticity of substitution, \( \sigma \), is defined as

\[
\sigma = \frac{\% \Delta \frac{x_2}{x_1}}{\% \Delta \frac{f_1}{f_2}} = \frac{\frac{x_2}{x_1}}{\frac{f_1}{f_2}} \times \frac{d \frac{f_1}{f_2}}{\frac{x_2}{x_1}}
\]

A graphic interpretation of this is helpful:

Can you think of examples of production inputs that are highly substitutable and some that are not?

**Short-Run Production**

In reality, the nature of production is such that over reasonable time horizons firms commit to certain production technologies that use some inputs in relatively fixed proportions. We summarize this idea in the short-run production and cost functions. Thus, in production theory, economists distinguish between the long-run, a planning period during which any aspect of a production process may be changed, and the short-run, a planning period during which there is at least one fixed factor of production. The existence of a fixed factor gives rise to diminishing marginal returns to the variable inputs, which, under certain conditions, in turn contributes to rising marginal costs. So, what determines the “shape” of \( c(q) \) in the short run? Two things:

- The nature of the underlying production process – the production function, \( f(x_1, x_2, \ldots, x_n) \).
- The behavior of the input prices, \( v_1, v_2, \ldots, v_n \).
The figures below summarize the essential features of short-run production theory.

It is common to divide an entire feasible range of short-run production into 3 stages. In **Stage 1** the variable input is being used with increasing efficiency. This stage continues until average product, or average output per unit reaches a maximum. Because the output per unit of the variable input is improving throughout stage 1, a price-taking firm will always operate beyond this stage.

In **Stage 2**, output increases at a decreasing rate, and the average and marginal product are both declining. Nevertheless, the average product of fixed inputs is still rising, because output is rising while fixed input usage is constant. In this stage, the employment of additional variable inputs increases the output per unit of fixed input but decreases the output per unit of the variable input. The optimum input/output combination for the price-taking firm will be in stage 2. It is important to note at this point, however, that a firm facing a downward-sloped demand curve (a “price maker”) might find it most profitable to operate in Stage 1.

In **Stage 3**, too much variable input is being used relative to the available fixed inputs: there are negative marginal returns to the variable input (and marginal costs rise exponentially). The output per unit of both fixed and variable inputs declines throughout this stage. At the boundary between stage 2 and stage 3, a point known as the **extensive margin**, the highest possible output is being obtained from the fixed input.

![Diagram of production stages](image)

Can you think of production settings in which diminishing marginal returns are not important? What implications would there be for cost behavior over a wide range of output?

**Relevant Textbook Problems:** 7.1, 7.2, 7.3, 7.5, 7.6, 7.7, 7.8, 7.9