

2.3: Structural Oligopoly Models

In this part of the class we will examine four formal models of oligopoly:

- Cournot
- Stackelberg (Leader-Follower)
- Bertrand

Key concepts:

Duopoly

Reaction functions

Cournot competition is a simultaneous game in which the players' key decision variable is quantity. A key concept: Cournot limit theorem

Stackelberg competition is a sequential game in which the players' key decision variable is quantity.

Bertrand competition may be thought of as a simultaneous game in which the players' key decision variable is price. Key concepts:

- Capacity constraints
- Dynamic competition
- Product differentiation

The efficiency of Cournot competition converges to that of perfect competition as the number of firms goes to infinity (the Cournot limit theorem). But how quickly might this convergence occur?

Relevant exercises: Problem Set 2, exercises 4 and 5.

Cournot Duopoly

- ❑ Two competitors simultaneously choose their output, each assuming that the other behaves just as they do.
- ❑ The solution may be conceived of as the simultaneous solution of each competitor's profit-maximizing reaction function.
- ❑ Example: Given $Q = a - bP$, $Q = q_1 + q_2$, and $c_1 = c_2$

$$q_1^* = \left(\frac{a - c_1}{2b} \right) - \frac{1}{2} (q_2)$$

$$q_2^* = \left(\frac{a - c_2}{2b} \right) - \frac{1}{2} (q_1)$$

$$q_1^* = \left(\frac{1}{3} \right) \left(\frac{a - c_1}{b} \right), \quad q_2^* = \left(\frac{1}{3} \right) \left(\frac{a - c_2}{b} \right)$$

and

$$Q^* = \left(\frac{2}{3} \right) \left(\frac{a - c}{b} \right), \text{ because } c_1 = c_2 \xrightarrow{\text{Generalizing for } n \text{ identical competitors}} Q^* = \left(\frac{n}{n+1} \right) \left(\frac{a - c}{b} \right)$$

Stackelberg Leader-Follower

- ❑ One competitor chooses its optimal output, assuming that the other competitors will respond in a manner predicted by their reaction functions.
- ❑ The solution may be conceived of as the *sequential* solution of each competitor's profit function:

Step 1: Leader (firm 1) maximizes profit, assuming follower (firm 2) responds by staying on its reaction function

$$\max_{q_1} \pi_1(q_1)$$

$$q_2^* = \left(\frac{a - c_2}{2b} \right) - \frac{1}{2} (q_1)$$
Step 2: Follower (firm 2) takes firm 1's output as given

$$q_1^* = \left(\frac{2}{3} \right) \left(\frac{a - c_1}{b} \right), \quad q_2^* = \left(\frac{1}{3} \right) \left(\frac{a - c_2}{b} \right)$$

and

$$Q^* = \left(\frac{2}{3} \right) \left(\frac{a - c}{b} \right)$$

Bertrand Duopoly

- ❑ Two competitors simultaneously choose their price, each assuming that the other behaves just as they do.
- ❑ The solution may be conceived of as the simultaneous solution of each competitor's profit-maximizing reaction function.

Assumptions:

1. Homogeneous good.
2. Consumers purchase from the low-price firm.

$$D_1(p_1, p_2) = \begin{cases} D(p_1) & \text{if } p_1 < p_2 \\ \frac{1}{2} D(p_1) & \text{if } p_1 = p_2 \\ 0 & \text{if } p_1 > p_2 \end{cases}$$

$$D_2(p_1, p_2) = \begin{cases} D(p_2) & \text{if } p_2 < p_1 \\ \frac{1}{2} D(p_2) & \text{if } p_1 = p_2 \\ 0 & \text{if } p_2 > p_1 \end{cases}$$

There are four possible price configurations:

1. $p_1 > p_2 < c$: This is not an equilibrium. Firm 1 could profitably deviate by setting $p_1 = p_2 - \epsilon$.
2. $p_1 > p_2 = c$: This is not an equilibrium. Firm 2 captures the entire market, but its profits are zero. Firm 1 could profitably deviate by setting $p_1 = p_2 - \epsilon$.
3. $p_1 = p_2 < c$: This is not an equilibrium since either firm could profitably deviate by setting $p_i = p_j - \epsilon$.
4. $p_1 = p_2 = c$: This is a Nash equilibrium. Neither firm can profitably deviate and earn greater profits even though profits are zero.

