

## 2.2: Repeated and Sequential Play in Game Theory

Repeated play may fundamentally alter the outcomes of a game.

- Reciprocity or retaliation must be a concern for both players.
- The payoffs are now multi-period and must be evaluated in terms of present value.

The “tit-for-tat” strategy is a common way to think about these implications.

The key question for each play amounts to this cost-benefit calculation: Does the present value of cooperation outweigh the present value of defection?

$$\sum_{t=0}^{\infty} \frac{\pi^K}{(1+\delta)^t} > (\pi^A - \pi^K)_{t=0} + \sum_{t=1}^{\infty} \frac{\pi^C}{(1+\delta)^t}$$

In other words, what may, upon first glance, appear to be collusive behavior, might in reality be the outcome of both players maximizing an objective function over time!

Because players can react to other players’ past actions, repeated games allow for equilibrium outcomes that would not be an equilibrium in the corresponding one-shot game.

Sequential games / key concepts:

- Backward induction
- Subgames
- Subgame perfect Nash Equilibrium

The credibility of threats are best analyzed in this way. What makes a threat credible?

**Repeated Play / Reciprocity and Supportable Equilibrium**

Suppose that player A adopts the following strategy:  
 Play a2. If Player B plays b2, continue to play a2. If Player B plays b1, then play a1 until Player B changes.

		B	
		b1	b2
A	a1	7, 4	9, 3
	a2	6, 11	8, 10

**Repeated Play / Reciprocity and Supportable Equilibrium**

Suppose that player A adopts the following strategy:  
 Play a2. If Player B plays b2, continue to play a2. If Player B plays b1, then play a1 until Player B changes.

Such a strategy will mean that both players face two possible streams of payoffs:

Time, t

	t <sub>1</sub>	t <sub>1</sub>	t <sub>1</sub>	...	t <sub>n</sub>	∞
Strategy 1: Defection	9	7	7	...	7	
Strategy 2: Cooperation	8	8	8	...	8	

**Repeated Play / Reciprocity and Supportable Equilibrium**

Define:  $\pi^K$  = Payoff from cooperation  
 $\pi^A$  = Payoff from defection  
 $\pi^0$  = Payoff from “being defected upon”  
 $\pi^C$  = Payoff from NE (Cournot competition)

		B	
		b1	b2
A	a1	$\pi_A^C, \pi_B^C$	$\pi_A^A, \pi_B^0$
	a2	$\pi_A^0, \pi_B^A$	$\pi_A^K, \pi_B^K$

**Sequential Games / Backward Induction & Subgame Perfect NE**

□ For each subgame, determine the optimal strategy

□ Find the optimal strategy for the “pruned” tree

Relevant exercises: Problem Set 2, exercises 2 and 3.