

Review Problems for Test #4

1. A cylindrical tank of radius 2 feet and height 8 feet sits with a circular end on the ground. It is initially partially filled with water to a depth of 5 feet. Find the work required to pump all the water in the tank to the top rim of the tank.
2. Suppose that a force of 15 pounds is required to stretch a spring 3 feet beyond its natural length. Compute the work required to stretch the spring from its natural length to 5 feet beyond its natural length. Assume Hooke's Law.
3. A certain satellite weighs 3 tons when it is on the surface of the earth. It follows from some fundamental laws of physics that the work (in mile-tons) required to lift this satellite from the earth's surface to an "infinite distance" above the earth's surface is given by the improper integral $\int_{4000}^{\infty} \frac{48 \times 10^6}{x^2} dx$. This value is sometimes called the "work required to escape the earth's gravity." Find this value, showing all "by-hands" steps needed to do this.
4. Show all "by-hands steps" involved in finding each of these indefinite integrals.

(a) $\int e^{\sqrt{x}} dx$

(b) $\int \frac{6}{x^3 + x^2 - 2x} dx$

(c) $\int \sin^3 x \cos^8 x dx$

5. Suppose that $f(x)$ is a function whose graph passes through the points (1, 4) and (2, 7) and that $\int_1^2 xf(x)dx = 17$. Use integration by parts to

evaluate $\int_1^2 x^2 f'(x)dx$

6. (a) Show all “by-hands steps” involved in using integration by parts to find this indefinite integral. Use the choice of u shown.

$$\int x^3 e^{x^2} dx$$

$$u = x^2$$

$$dv =$$

- (b) Show all “by-hands steps” involved in using partial fractions to find this integral of a rational function.

$$\int \frac{6x+39}{x^2+11x+28} dx$$

7. (a) Use Maple’s “convert(expression,parfrac,x)” command to determine the partial fractions form for the rational function $\frac{1}{x^4-1}$.

- (b) Evaluate the indefinite integral $\int \frac{1}{x^4-1} dx$ by integrating by hand the partial fractions form for the integrand which you found in part (a).

8. Explain why $\int_0^2 \frac{e^{\sqrt{2-x}}}{\sqrt{4-x^2}} dx$ is an improper integral, and then write it as the appropriate limit of a proper integral. Use Maple to evaluate the improper integral directly. Is the improper integral convergent or divergent?

9. (a) Write the improper integral $\int_3^5 \frac{1}{\sqrt{x-3}} dx$ as an appropriate limit, and then find its value.

- (b) Write the improper integral $\int_{-\infty}^{-2} \frac{1}{x^5} dx$ as an appropriate limit, and then find its value.

10. (a) Use l’Hopital’s rule to find $\lim_{x \rightarrow 0} \frac{\arcsin(2x)}{x}$.

- (b) Use l’Hopital’s rule to find $\lim_{n \rightarrow \infty} n^2 \sin\left(\frac{1}{n}\right)$

11. For the sequence $a_n = \frac{2^{n+1}}{n!}$ ($n \geq 1$)
- (a) Write out the values of the first four terms of the sequence.
 - (b) Write out the values of the first four partial sums.
12. Determine convergence or divergence of each of the following series. If the series diverges, explain why. If the series converges, show steps to find the sum.

(a)
$$\sum_{n=0}^{\infty} \frac{6}{5^n}$$

(b)
$$\sum_{n=0}^{\infty} 7\left(\frac{\pi}{4}\right)^n$$

(c)
$$\sum_{n=1}^{\infty} \left(\frac{n^3-1}{2n^3+1}\right)$$