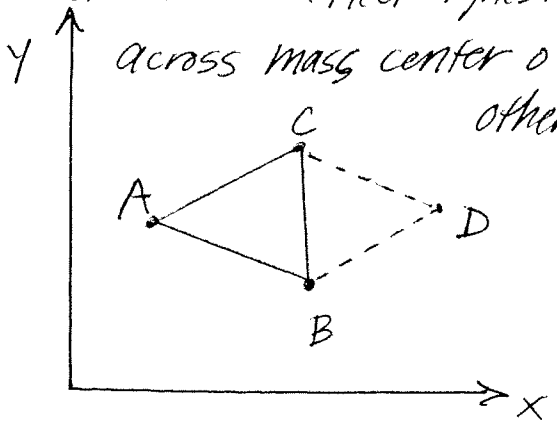
Nelder-Mead algorithm:

ref. Haupt & Haupt.

Sort guesses s.t. $f(A) > f(B) > f(C)$

1. Reflection - reflect highest pt. across mass center of others



$D = B + C - A$

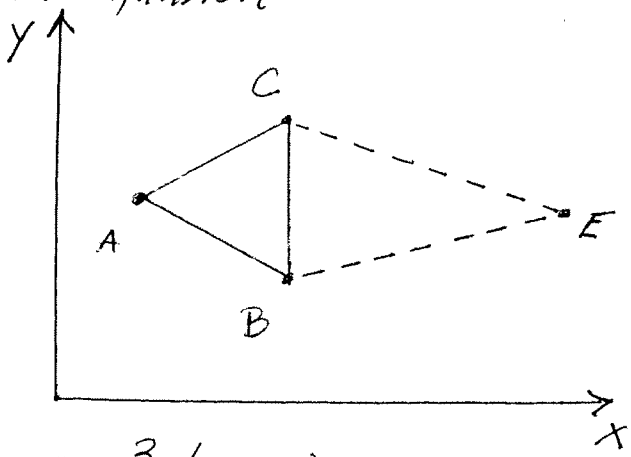
If $f(B) > f(D) > f(C)$

$A = D$ & repeat 1.

else if $f(B) > f(C) > f(D)$

go to 2.

2. Expansion



$E = \frac{3}{2}(B+C) - 2A$

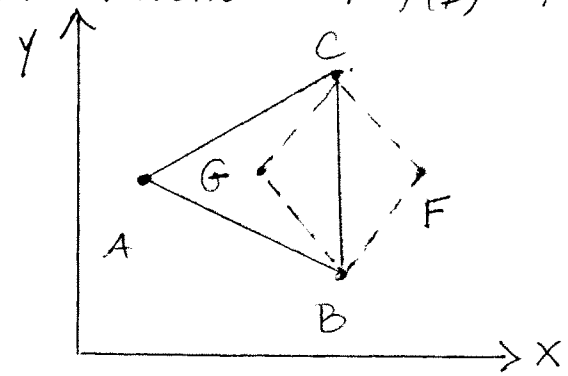
If $f(E) < f(D)$

$A = E$ & repeat 1.

else

$A = D$ & repeat 1

3. Contraction if $f(D) > f(B)$



$G = \frac{1}{4}(B+C) + \frac{1}{2}A$

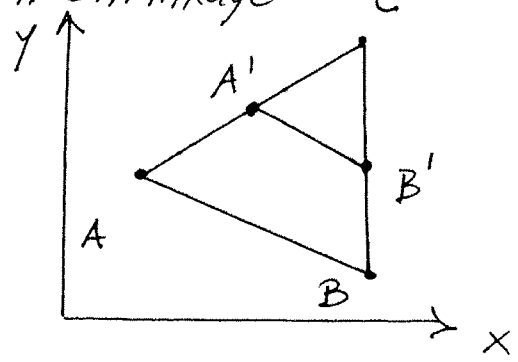
$F = \frac{3}{4}(B+C) - \frac{1}{2}A$

If $f(F) < f(A)$ | $f(G) < f(A)$

$A = \min(F, G)$, repeat 1

else go to 4

4. Shrinkage:



$$A' = \frac{A+C}{2}$$

$$B' = \frac{B+C}{2}$$

Notes:

- In 2D, the simplex is a triangle. In 3D it is a tetrahedron, etc.
- For nD, the algorithm steps are modified as follows: (Wikipedia)

0. Sort

$$f(\underline{x}_1) < f(\underline{x}_2) < \cdots < f(\underline{x}_{n+1})$$

$$\underline{x}_0 = \frac{1}{n}\sum_{i=1}^{n} \vec{x}_i$$

(compute mass center)

1. Reflect about mass center:

$$\underline{x}_r = \underline{x}_0 + \alpha(\underline{x}_0 - \underline{x}_{n+1})$$

2. Expansion

$$\underline{x}_E = \underline{x}_0 + \gamma(\underline{x}_0 - \underline{x}_{n+1})$$

3. Contraction

$$\underline{x}_F = \frac{\underline{x}_0}{\rho} - \underline{x}_G$$

$$\underline{x}_G = \rho\underline{x}_0 + \rho\underline{x}_{n+1}$$

4. Shrinkage

$$\underline{x}_i = \sigma(\underline{x}_i + \underline{x}_1) \quad \forall i \neq 1$$

With coefficients

$\alpha = 1$; $\gamma = 2$

$\rho = \frac{1}{2}$; $\sigma = \frac{1}{2}$

And all logic is unchanged with $A = \underline{x}_{n+1}$

$C = \underline{x}_1$

$B = \underline{x}_n$