1. Let $E \subset (-\pi, \pi)$ with $m(E) > 0$. Fix $\delta > 0$. Show that there are at most finitely many integers n such that $\inf_{x \in E} \sin(nx) \geq \delta$. Hint: Bessel.

2. Show that for any fixed function $g \in L^2(0, \pi)$ we have

$$\lim_{k \to \infty} \int_0^\pi g(x) \sin(kx)\, dx = 0.$$

3. Let H be a Hilbert space (say over the reals, for simplicity), f an element of H, and f_k a sequence in H with the property that $< f_k, g >$ converges to $< f, g >$ in $\mathbb{R}$ for each fixed $g \in H$.

   (a) Show that f_k need not converge to anything in H. Hint: The last problem.

   (b) Show that if in addition to $< f_k, g > \to < f, g >$ for each g we also have $\|f_k\| \to \|f\|$, then $f_k \to f$ in H.