Homework 3

MA430

As always, give full justification for your answers!

1. Read Sections 3.1 to 3.5 in the text.

2. Let $\mathbf{x} =< -2, 0, 1, 3, 0, 0 >$ be a vector in $\mathbb{R}^6$.

   (a) Is $\mathbf{x}$ 1-sparse?, 2-sparse? 3-sparse?

   (b) Compute $\text{supp}(\mathbf{x})$ and $\|\mathbf{x}\|_0$.

   (c) If $T = \{2, 3, 5\}$, what is $\mathbf{x}_T$?

3. Let $\mathbf{x}$ be as in the last problem, but take $T = \text{supp}(\mathbf{x})$. Let

$$\mathbf{A} = \begin{bmatrix} 1 & 1 & 1 & 1 & -1 & -1 \\ 0 & 2 & 2 & -3 & 1 & 4 \end{bmatrix}.$$

   Compute $\mathbf{A}_T$ and $\mathbf{x}_T$. Verify that $\mathbf{A}\mathbf{x} = \mathbf{A}_T\mathbf{x}_T$!

4. Let

$$\mathbf{A} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ -1 & 2 & -2 & 1 \end{bmatrix}.$$

   Show that $N(\mathbf{A})$ contains no (nonzero) 2-sparse vectors. You can do this by showing that any vector of the form $\mathbf{x} =< x_1, x_2, 0, 0 >$ that satisfies $\mathbf{A}\mathbf{x} = \mathbf{0}$ must be $\mathbf{x} = \mathbf{0}$. Then try $\mathbf{x} =< x_1, 0, x_3, 0 >$, then $\mathbf{x} =< x_1, 0, 0, x_4 >$, etc. (six possible combinations). Use Maple. Still tedious, isn't it?

5. Suppose I have 10 marbles, nominal mass 10 grams each, but a few marbles may be defective and have mass that's slightly off. Let x_i denote the deviation of the ith marble from nominal. I weigh a subset consisting of marbles $\{3, 6, 10\}$ and get 30.13. I can conclude that

$$x_3 + x_6 + x_{10} = 0.13$$

   (the set had mass 30.13 grams, had 3 marbles, an excess of 0.13 grams). Suppose I weigh additional subsets consisting of marbles $\{1, 4, 6, 7\}$, $\{1, 2, 8, 9, 10\}$, $\{2, 3, 4, 5, 8, 10\}$, $\{1, 2, 5, 6, 7\}$, and $\{3, 7, 10\}$, and obtain masses $40.00, 49.63, 59.76$, 50.00 and 30.00 grams, respectively.

(a) Write out the other five relevant equations and formulate this as a system $\mathbf{Ax} = \mathbf{b}$, where $\mathbf{A}$ is 6×10.

(b) What is coherence of $\mathbf{A}$? What sparsity solutions are guaranteed to be unique?

(c) Find a minimal norm solution to $\mathbf{Ax} = \mathbf{b}$. What is its sparsity?

6. Make up 100 random matrices with standard normal random variables as entries (use either Maple or Matlab, I'll supply some template code) of size $m \times n$ for the values of m and n listed below. In each case compute the average coherence of the matrices.

m	n	ave coherence
5	10	
5	20	
5	100	
10	10	
10	20	
10	50	
20	50	

What is the general behavior of the coherence with respect to m and n? Does that make sense?

7. Make up a 2×4 matrix $\mathbf{A}$ with the smallest possible coherence. Hint: coherence is is a measure of the "angle" between the columns of $\mathbf{A}$, so think geometrically. What is the coherence your matrix?