Straightedge and Compass II

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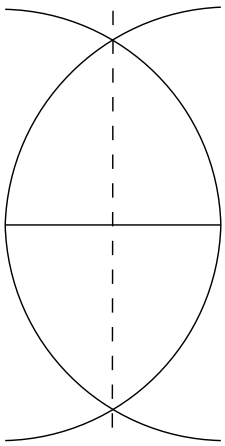
NAME:_____

1. **Review**

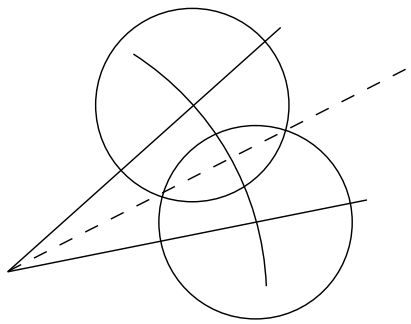
   Remember the three rules for drawing with the straightedge and compass:

   - We can use the straightedge to draw a line though any two marked points, or
   - Use the compass to draw a circle centered at any marked point and passing though any other point, or
   - Mark the points at which any lines, circles, or circle and lines intersect.
   - We cannot "eyeball" anything!

   The figures below might help you remember how we accomplished a few tasks last week, using only a straightedge and compass:



Bisecting a Line Segment



Bisecting an Angle

2. **Drawing Perpendicular Lines**

   (a) On a fresh sheet of paper, draw a line. Make it go to the edges of the paper. Label this line "L".

   (b) Mark a point on the paper that's not on the line, say about one inch away from the line. Label this point "A". **Our goal is to draw a line through A which is perpendicular to the line L.**

   (c) Set your compass so the tips are farther apart than the distance from A to the line L and draw a circle centered at A. It should intersect L in two points, which you should label "B" and "C".

   (d) Use your compass to draw circles (of the same radius as in part (c)) through B and C. The circles should both pass through A, and also intersect at another point "D" (label it!) on the other side of the line L. If you connect A and D, you'll get the perpendicular line we want! The point D is the "reflection" of A over L.

(e) Connect the original point A to the point D with a line segment. This line segment passes through A and is perpendicular to the line L.

3. **Drawing Parallel Lines**

   (a) Let's re-use the last drawing. **Our goal is to construct a line through A which is PARALLEL to L**. Label the point where the line L and the perpendicular segment intersect each other as "P".

   (b) Put the non-drawing tip of your compass on P and the drawing tip on the point A. Draw a circle (P should be at the center, and A on the circle).

   (c) The circle should intersect L at two points. Label one of them "Q". Put the non-drawing tip of your compass on Q and the drawing tip on the point P. Draw a circle (Q should be at the center, and P on the circle).

   (d) Put the non-drawing tip of your compass on A and the drawing tip on the point P. Draw a circle (A should be at the center, and P on the circle).

   (e) The circles from parts (c) and (d) should intersection at two points—P, and another point. Label the other point "R". Now draw a line through A and R. That line should be parallel to L!

4. **Drawing a Square** This is easy—connect the points Q and R in the last drawing and you'll have a perfect square!

5. **Drawing a Hexagon** A hexagon has six sides. We're going to draw a "regular hexagon", meaning all sides have the same length.

   (a) Start by drawing an equilateral triangle like we did last week: Pick two points, labelled "A" and "B". Connect them with a line segment. Then put the non-drawing tip of the compass at A, the drawing tip at B, and draw a circle. Now put the non-drawing tip of the compass at B, the drawing tip at A, and draw a circle. The circles should intersect at two points. Mark one and label it "C". Connect A to C and B to C to get an equilateral triangle.

   (b) Bisect each angle of the equilateral triangle. All the bisecting lines should meet in the "center" of the triangle. Label this point "P".

   (c) Put the non-drawing tip of the compass on P, the drawing tip on any of A, B, or C, and draw a circle. The circle intersects the each of the lines which bisect the angles of the equilateral triangle. Mark these points. Do you see how to connect everything to make a regular hexagon?

6. **A Challenge**: Can you figure out how to take the square in problem (4) and turn it into a regular octagon (eight sides)? Hint: proceed like we did with the hexagon.

7. **Other Regular Polygons** It turns out you can also draw a regular pentagon (five sides). A mathematician named Carl Friedrich Gauss (the greatest mathematician of the 19th century) showed that one can use a straightedge and compass to draw a regular 17-gon, and also a regular 65,537-gon! (He didn't actually draw them, though).

However, Gauss also proved that some shapes CANNOT be constructed with a straight-edge and compass (if you stick to the three rules we discussed). In particular, it's impossible to draw a regular heptagon (seven sides).