

*The Unreasonable Effectiveness
of Mathematics*



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Overview

- Unreasonable Effectiveness??
- Example 1 Fast Fourier Transform
- Example 2 Scientific Visualization and the Traveling Salesman Problem
- Other examples - a list
- What it means to Rose Mathematics

Unreasonable Effectiveness??

- **Mathematical models, mathematical thinking and mathematical abstractions have a power far beyond what might reasonably be expected**
- Why?
- mathematics is the pervasive language of science and engineering
- mathematics has power for conceptualization, computation, and application
- the introduction of the computer has greatly magnified this power
- total mathematical knowledge is being invented or discovered at an exponential rate

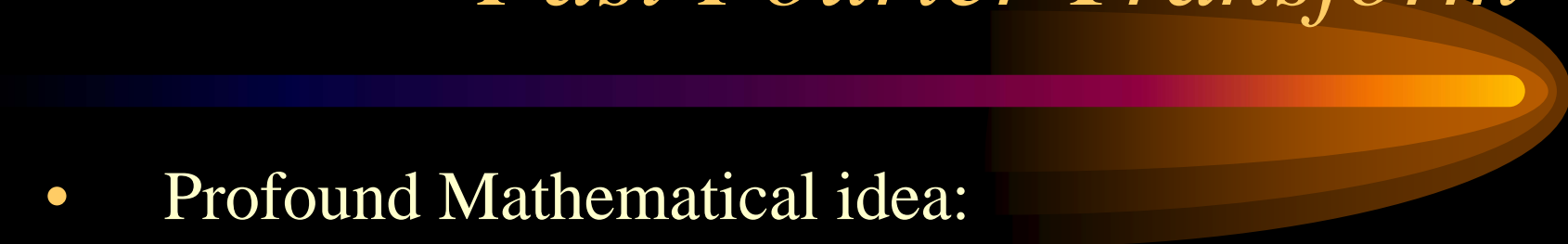
Example 1- p1

Fast Fourier Transform

- Basic idea Fourier approximation about 200 years old
- Eliminating noise is a simple problem solved by Fourier methods
- Discretize (sample) and use the Discrete Fourier Transform, a more modern concept
- Matlab Show (dftdemo.m)

Example 1 - p2

Fast Fourier Transform



- Profound Mathematical idea:
 1. transform
 2. do something
 3. come back
- But another great math idea comes into play

Example 1 - p3

Fast Fourier Transform

- DFT slow $O(n^2)$
- FFT by Cooley-Tukey algorithm is fast $O(n \log(n))$
- clock arithmetic (group theory) and prime factorization are the abstract enabling concepts sitting in the background
- Matlab show (fftperform.m)

Example 1 - p4

Fast Fourier Transform

- Cooley -Tukey algorithm changes an interesting theory into a powerful tool
- Impact of mathematics in DSP and imaging continues with wavelet transform
- JPEG2000

Example 2 - p1

Scientific Visualization and TSP



- Use an image to visualize relations among a set of parameters or other classes
- Matlab show (SVisTSP.m)
- Show Discrete Grid (grid.pdf)

Example 2 - p1

Scientific Visualization and TSP

- 3x3 case has a discrete grid of 36 points
- 20x20 case has discrete grid of 5.9×10^{36} points
- Time to call Dave Rader!

More Examples

- RSA & Elliptic Curves cryptography -
Clock Arithmetic & Group Theory
- Internet Traffic and Linear Algebra
- Grand Unified Theories and String Theory
- Differential Geometry and Cosmology
- Data Mining Statistical models
- Mapping the Human Genome

What it means to Rose Mathematics-Faculty

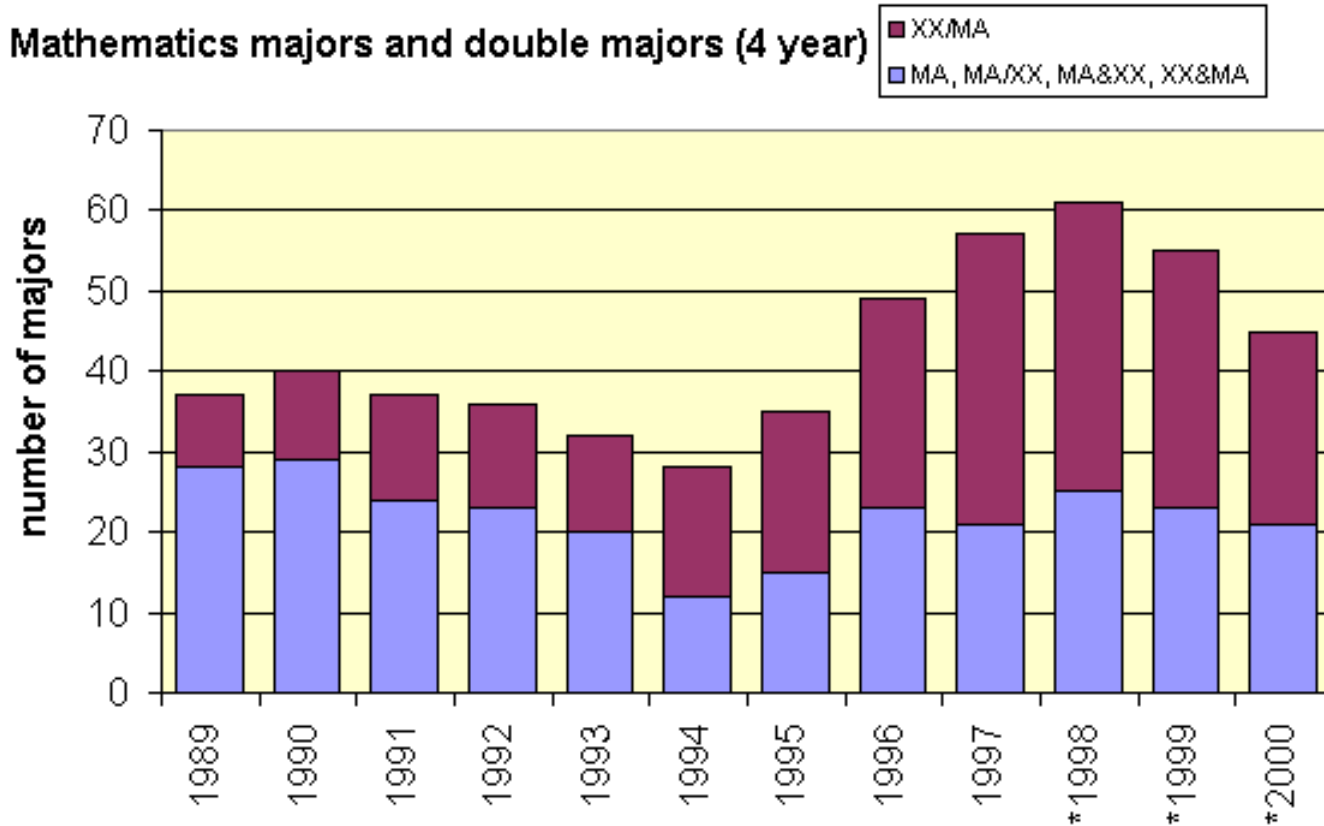
- Diverse areas of interest, not just areas thought to traditionally support engineering and science
- Creative people who are willing to think and work in diverse areas of mathematics and diverse areas beyond mathematics
- Use computing wisely know when and how to compute and know when to think mathematically
- Faculty who engage students with their interest in mathematics

What it means to Rose Mathematics-Students

- All students should become competent users of mathematics
- mathematics is an excellent vehicle for developing problems solving skills in all students
- mathematics is an excellent vehicle for developing skillful computer use in all students
- for the many students who are interested mathematics provides an exciting and challenging enhancement to their studies and a career prospects
- projects and undergraduate research are a good way to engage them

Math Majors

Mathematics majors and double majors (4 year)



What it means to Rose Mathematics-Programs

- Continue to develop Discrete Mathematics and Statistics programs
- Continue broad education, math major as problem solver, for industry bound students
- Continue to emphasize, strongly, double majors and minors

Thank You



- Thanks to the Institute for its confidence it has shown in me.
- Thanks to my colleagues in the department.
- It is a pleasure to work with them.