Wavelet Based Methods in Image Processing

Lecture 2 - Filtering and Convolution

Applied Mathematics Seminar

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Lecture 2 & 2.a

- lecture 2
  - smoothing and differencing (filtering)
  - Maple and Matlab demos

- lecture 2.a
  - frequency domain
  - Fourier transform
  - convolution theorem
  - restoration
Smoothing and differencing

- smoothing
  - eliminate high frequency detail
  - let low frequency structure pass through
  - low pass filtering

- differencing
  - eliminate low frequency structure
  - let high frequency structure pass through
  - high pass filtering
Maple: filtering of signals

- graphs
- algebra
- low pass \( X \rightarrow H_l X = l * X \)
- high pass \( X \rightarrow H_h X = h * X \)
- filter1d.mws
Vector Convolution - 1D

- \( X = [X(0),...,X(N-1)] \) vector of length \( N \)
- \( g \), vector of possibly smaller length, zero pad out to size \( N \)

\[
g \ast X(r) = \sum_{s=0}^{N-1} g(r - s) X(s)
\]

- computation \( \text{mod} \ N \)
Matlab: filtering of images

- horizontal blur \[ X \rightarrow H_lX = l^\ast X \]
- vertical blur \[ X \rightarrow XH_l^t = X \ast l^t \]
- bi-directional blur \[ X \rightarrow H_lXH_l^t = l^\ast X \ast l^t \]
- horizontal details \[ X \rightarrow H_hX = h^\ast X \]
- vertical details \[ X \rightarrow XH_h^t = X \ast h^t \]
- diagonal details \[ X \rightarrow H_hXH_h^t = h^\ast X \ast h^t \]
- blur.m, edgedet.m, ansmid3.m
Matrix Convolution - 2D

- $X = [X(i,j)]$ $m \times n$ matrix
- $M$, convolution mask matrix of possibly smaller size, zero pad out to size $m \times n$

$$M \ast X(k,l) = \sum_{r=0}^{m-1} \sum_{s=0}^{n-1} M(k-r,l-s)X(r,s)$$

$$= \sum_{r=0}^{m-1} \sum_{s=0}^{n-1} X(k-r,l-s)M(r,s)$$

- computation $mod \ m$ in $r,k$, $mod \ m$ in $s,l$