

Do Multiproduct Firms with Market Power Cross-Subsidize?

by

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I. Introduction

Firms with multiple products and market power are often alleged to charge higher prices on some of their products to cover the high cost they incur in producing other products. There are numerous anecdotal examples such as the argument that colleges allegedly cross-subsidize the cost of graduate education with the monies from undergraduate education. Paper producers claim mandatory recycling of their products is used to cross-subsidize the less profitable recycling of cans and bottles. The high stamp prices in Canada supposedly cross-subsidizes other Canada Post activities. Medical centers may use research monies to subsidize the cost of medical education. This paper uses a theoretical model to investigate whether a multiproduct monopolist cross-subsidizes the higher cost of one product by charging higher prices for an unrelated product.

Using a simple model with linear demand curves and constant marginal costs, this paper demonstrates that a two-good monopolist facing demand curves that are known with certainty will not increase the price of one good because of an increase in the cost of producing another, unrelated good. With certain demand, a monopolist will not cross-subsidize the increase in the cost of one good by charging a higher price on another good. However, once uncertainty is introduced, the paper shows that a monopolist that maximizes expected utility will increase the price of the good facing uncertain demand given an increase in the cost of an unrelated good which faces a certain demand curve. So it is in the face of uncertainty that the monopolist will charge higher prices for some goods in response to higher cost for another good. However, if the

cost of producing the good with uncertain demand increases, the price of the good with certain demand is not affected.

Following these introductory comments is a brief review of the literature upon which the cross-subsidization model developed in this paper is based. The paper's third section presents two models - - one where both of the monopolist's products faces certain demand and another where one of the products faces uncertain demand. Conclusions and thoughts about future research are discussed in the fourth and final section of the paper.

II. Review of the Literature

This paper's genesis has roots in three branches of economic literature: the issues concerning price theory and uncertainty, the output-price decisions of a multiproduct monopolist, and the cross-subsidization literature. These different areas of the literature are discussed in that order.

Price theory and uncertainty

Mills (1959) analyzed the effect of uncertainty on a monopolist's pricing decisions, finding the results depends on the shape of the monopolist's marginal cost curve. For example, in the case of constant marginal cost, Mills determined that the monopolist's optimal price will be lower with uncertainty than without. In a dynamic theory that emphasizes the role of inventories of the finished product, Zabel (1970) also examined the pricing behavior of a monopoly with uncertain demand. He demonstrates that the monopolist's optimal level of inventory will fall as the holding cost of inventory increases. Given a uniform distribution of demand and constant marginal cost, Zabel finds a monopoly will increase its price as the holding cost of inventory also increases.

In an often-cited work, Sandmo (1971) investigates the output decisions of a competitive firm under price uncertainty.¹ He finds two key facts. First, under price uncertainty, the output of the competitive firm is less than the output that would occur with price certainty. Secondly, if decreasing absolute Arrow-Pratt risk aversion is assumed,² then a competitive firm with price uncertainty will reduce output as its fixed cost increases. Leland (1972) extends Sandmo's result to the theory of monopoly under uncertainty. He finds that under uncertainty a monopolist's price and output decision are not invariant to changes in fixed cost. If the demand curve exhibits what Leland calls the "principle of increasing uncertainty," then the quantity-setting monopolist will produce a smaller output than the certainty amount where a known marginal revenue curve intersects the marginal cost curve.

Harris and Raviv (1981) use a model of demand uncertainty, and they find that endogenously derived pricing schemes for a monopolist depend on capacity constraints. In their results, an optimal single price exists only if capacity constraints are not binding. Using a capital asset pricing model that explicitly includes risk, Brick and Jagpal (1981) examine a monopoly's decisions regarding price and advertising under uncertainty. Not surprising, they find increases in demand leads to increases in the monopoly price. The optimal level of advertising, however, depends on how responsive the risk-adjusted price elasticity of demand is to changes in advertising.

Multiproduct firms

One of the first papers to study the implications of uncertain demand on a multiproduct monopoly was the work done by Dhrymes (1964). He decomposed this problem into two components. First, the monopolist determines the optimal output mix by maximizing expected

¹ This material was further debated in an exchange between Sandmo (1972) and Bernhardt (1972).

² This concept will be discussed more fully in Section III.

utility. Next, given this optimal output mix, the monopoly determines the optimal combination of inputs by minimizing cost. Dhrymes concludes that the qualitative results of his model are similar to those of a uniproduct firm; but, the multiproduct firm's response to changes in the state of the uncertainty is more complex than that of a uniproduct firm. Exogenous shocks to the state of uncertainty include both changes in the firm's attitude toward risk and changes in the underlying probability distribution function that characterizes the firm's risk.

Meyer (1975, 1976) extends the analysis of a monopoly under uncertainty to a monopoly with multiple outputs and multiple inputs. In his 1975 paper dealing with simultaneous pricing and capacity decisions under uncertainty, Meyer found that the optimal investment decision usually entailed some excess capacity. His 1976 paper applied components of the capital asset pricing theory to directly incorporate the market price of risk. In this paper, Meyer found that the optimal pricing structure depended on the marginal risk associated with each distinct group of customers. One interesting result was his finding that optimal pricing may involve selling output to several groups of customers at a price below the marginal production cost.

The cross-subsidization literature

When the economic literature refers to cross-subsidization, often it is in reference to the behavior of a regulated monopolist. Church and Ware (1999, p.797) state cross-subsidization exists "if the revenues from a product are less than its costs of production." An example would be ATT's cross-subsidization of low local rates with higher long-distance rates that occurred before its breakup in the 1980s. A current example of a regulated firm using cross-subsidization would be Palmer's (1992) conclusion that local businesses in New England subsidized residential phone rates. Faulhaber (1975) examines cross-subsidization in publicly owned

facilities - - a type of regulated monopoly - - from a game theoretic point of view. Shaffer (1984) examines cross-subsidization in checking accounts.

In these studies, cross-subsidization usually referred to the use of profits from one activity to cover the losses in another activity. This paper uses an expanded definition of cross-subsidization. The question here is whether multiproduct firms will increase the price of one product given an increase in the per unit cost of another, unrelated product.

III. The Model

Suppose a profit-maximizing, quantity-setting monopolist sells two products, good 1 and good 2, in separate markets.³ Symbolically, q_1 is the output of good 1, q_2 is the output of good 2, p_1 is the market price of good 1, and p_2 is the market price of good 2. The monopolist faces a linear demand curve in each market; so $p_1 = a_0 - bq_1$ and $p_2 = g_0 - hq_2$, where a_0 and g_0 are the positive vertical intercepts of the demand curves, and b and h are the absolute values of the slopes of the two demand curves. Each product is produced with constant per unit cost. The constant marginal cost of good 1 is c_1 , while c_2 is the constant marginal cost of good 2.

Goods 1 and 2 are assumed to be unrelated. Changes in the price of one good does not affect the demand for the other good.⁴ In addition, there are no synergies in production as c_1 and c_2 are unrelated.⁵ First, these assumptions are used in a model with complete certainty about the demand for both products. The results from this simple model with certainty are later compared to the outcome of another model where the demand for good 1 is uncertain.

³ Assume good 1 is sold in the first market or market 1, while good 2 is sold in the second market or market 2.

⁴ Goods 1 and 2 are neither substitutes or complements. In other words, $\partial p_1 / \partial p_2 = \partial p_2 / \partial p_1 = 0$.

⁵ It is assumed $\partial c_1 / \partial q_2 = \partial c_2 / \partial q_1 = 0$.

The case of certain demand

The monopolist chooses q_1 and q_2 to maximize profits, π , which equals

$$(1) \quad \pi(q_1, q_2) = p_1q_1 - c_1q_1 + p_2q_2 - c_2q_2 = (a_0 - c_1)q_1 - bq_1^2 + (g_0 - c_2)q_2 - hq_2^2 .$$

Differentiating equation (1) with respect to q_1 and q_2 results in two first-order conditions:

$$(2) \quad \frac{\partial \pi(q_1, q_2)}{\partial q_1} = a_0 - 2bq_1 - c_1 = 0$$

and

$$(3) \quad \frac{\partial \pi(q_1, q_2)}{\partial q_2} = g_0 - 2hq_2 - c_2 = 0.$$

The economic interpretations of equations (2) and (3) are straightforward: to maximize total profits, the monopoly will sell the amount of goods 1 and 2 that equate their marginal revenue to their marginal cost of the good.⁶

The profit-maximizing amount of each good, q_1^* and q_2^* , is found by solving equations (2) and (3) to find

$$(4) \quad q_1^* = \frac{a_0 - c_1}{2b}$$

and

$$(5) \quad q_2^* = \frac{g_0 - c_2}{2h} .$$

The key result from the model with certainty can be seen from equations (4) and (5). Given these optimal solutions, note that $\partial q_1^* / \partial c_2 = \partial q_2^* / \partial c_1 = 0$. In the case of certainty, the monopolist's profit-maximizing output in one market is invariant to the monopolist's per unit

⁶ The marginal revenue in the first market equals $a_0 - 2bq_1$ and $g_0 - 2hq_2$ is the marginal revenue in the second market.

cost in another, unrelated market. A change in c_2 will not affect the monopolist's profit-maximizing price for p_1 just as the firm's optimal choice of p_2 is invariant to a change in c_2 . This result is intuitive. If a monopolist is charging the profit-maximizing price for one product, it is common sense that the firm will not change this price given a change in the production cost of an unrelated product. Any deviation in the price of the good whose cost did not change from its initial profit-maximizing level will decrease total profits more than the initial increase in the other good's per unit cost. Thus, in this static model with certainty, a profit-maximizing, multiproduct monopolist does not cross-subsidize increases in the per unit cost of one good with increases in the price of an unrelated good that it also sells.

The case of uncertain demand

To introduce uncertainty in the model, good 1 is assumed to have uncertain demand. Suppose there are two states of the world. In the first state, State 1, which occurs with probability z_1 , the demand curve for good 1 is $p_1 = a_0 - bq_1$. State 2, the second state, occurs with probability z_2 ; however, the demand for good 1 is $p_1 = a_1 - bq_1$ where $a_1 > a_0$. Thus, uncertainty in the demand for good 1 is captured by an intercept shift. Profits in State 1, π_1 , equal the sum of the certain profits from the sale of good 2 and the profits from selling good 1 at the lower demand or

$$(6) \quad \pi_1 = (a_0 - bq_1)q_1 - c_1q_1 + (g_0 - hq_2)q_2 - c_2q_2 .$$

Likewise, the profits the monopoly earns in State 2, π_2 , equal the sum of the certain profits generated by the sale of good 2 and the profits from the sale of good 1 with the increased demand or

$$(7) \quad \pi_2 = (a_1 - bq_1)q_1 - c_1q_1 + (g_0 - hq_2)q_2 - c_2q_2 .$$

Given the uncertainty about the demand for good 1, the monopolist determines the optimal values of q_1 and q_2 by maximizing its expected utility function, $E(U) = z_1 U(\pi_1) + z_2 U(\pi_2)$, where $U(x)$ is the utility of income.⁷

Differentiating $E(U)$ with respect to q_1 and q_2 results in two first-order conditions or

$$(8) \quad \frac{\partial E(U)}{\partial q_1} = z_1 U'(\pi_1)(a_0 - 2bq_1 - c_1) + z_2 U'(\pi_2)(a_1 - 2bq_1 - c_1) = 0$$

and

$$(9) \quad \frac{\partial E(U)}{\partial q_2} = [z_1 U'(\pi_1) + z_2 U'(\pi_2)](g_0 - 2hq_2 - c_2) = 0 .$$

Given that both z_1 and z_2 are positive fractions, and the marginal utility of income, $U'(\pi_1)$ and $U'(\pi_2)$, are positive, then equation (9) implies that $(g_0 - 2hq_2 - c_2)$ equals 0. Thus, the multiproduct monopolist will produce that level of q_2 that equates the marginal revenue in the second market to the marginal cost in the second market. The monopolist produces the certainty output in the market where demand is certain. In other words, the optimal q_2^* resulting from equation (9) is the same level of q_2 resulting from equation (3) when the demand for both products is known with certainty, namely $q_2^* = (g_0 - c_2)/2h$. This result anticipates the comparative statics below that show the optimal level of q_2 is invariant to changes in c_1 .

Given the assumptions about the z_i and the $U'(\pi_i)$, $i = 1, 2$, equation (8) implies that the two terms - - $(a_0 - 2bq_1 - c_1)$ and $(a_1 - 2bq_1 - c_1)$ - - have opposite signs. Since $a_1 > a_0$, it follows that $(a_0 - 2bq_1 - c_1) < 0$ and $(a_1 - 2bq_1 - c_1) > 0$. Given the uncertain demand for good 1, the optimal level of q_1, q_1^* , must satisfy two conditions. First, at q_1^* the marginal revenue associated

⁷ Clearly, $0 < z_1 < 1$, $0 < z_2 < 1$, and $z_1 + z_2 = 1$. Additionally, the marginal utility of income, $U'(\pi_i)$, is assumed to be positive. Risk aversion would imply $U''(\pi_i) < 0$.

with the lesser demand for good 1 is less than the marginal cost of producing good 1.

Conversely, the marginal revenue associated with the greater demand for good 1 is greater than the marginal cost of good 1 at q_1^* . If q_1^\dagger is the profit-maximizing level of output of good 1 with the lesser demand and q_1^\ddagger is the profit-maximizing level of output of good 1 with the increased demand, then as Figure 1 shows, $q_1^\dagger < q_1^* < q_1^\ddagger$.⁸ Thus, given uncertain demand, the firm's optimal output of q_1 exceeds the profit-maximizing amount in the case of the lesser demand but is less than the profit-maximizing amount in the case of the greater demand.

The sufficient, second-order conditions of this optimization problem involve the second partial derivatives of the expected utility function, $E(U)$. In matrix form, these second partial derivatives are

$$(10) \quad H = \left[\frac{\partial^2 E(U)}{\partial q_i \partial q_j} \right] = \begin{bmatrix} H_{11} & 0 \\ 0 & H_{22} \end{bmatrix}$$

where

$$(11) \quad H_{11} = z_1 U''(\pi_1)(a_0 - 2bq_1 - c_1)^2 + z_2 U''(\pi_2)(a_1 - 2bq_1 - c_1)^2 - 2b[z_1 U'(\pi_1) + z_2 U'(\pi_2)]$$

and

$$(12) \quad H_{22} = -2h[z_1 U'(\pi_1) + z_2 U'(\pi_2)].$$

Maximizing $E(U)$ requires $H_{11} < 0$, $H_{22} < 0$, and the determinant of H , \bar{H} , which equals $H_{11}H_{22}$, must be positive. The off-diagonal terms of matrix H are zero because

$$\frac{\partial^2 E(U)}{\partial q_1 \partial q_2} = \frac{\partial^2 E(U)}{\partial q_2 \partial q_1} = [z_1 U''(\pi_1)(a_0 - 2bq_1 - c_1) + z_2 U''(\pi_2)(a_1 - 2bq_1 - c_1)](g_0 - 2hq_2 - c_2)$$

and $(g_0 - 2hq_2 - c_2) = 0$ because of the first order-condition in equation (9).

⁸ At q_1^\dagger , $a_0 - 2bq_1 = c_1$, while $a_1 - 2bq_1 = c_1$ at q_1^\ddagger .

Evaluating equations (8) and (9) at the solutions, $q_1^* = q_1^*(c_1, c_2)$ and $q_2^* = q_2^*(c_1, c_2)$, and differentiating both of these equations with respect to c_2 , a standard comparative statics exercise finds

$$(13) \quad \begin{bmatrix} H_{11} & 0 \\ 0 & H_{22} \end{bmatrix} \begin{bmatrix} \partial q_1^* / \partial c_2 \\ \partial q_2^* / \partial c_2 \end{bmatrix} = \begin{bmatrix} \theta_1 \\ \theta_2 \end{bmatrix}$$

where

$$(14) \quad \theta_1 = q_1 [z_1 U''(\pi_1)(a_0 - 2bq_1 - c_1) + z_2 U''(\pi_2)(a_1 - 2bq_1 - c_1)]$$

and

$$(15) \quad \theta_2 = z_1 U'(\pi_1) + z_2 U'(\pi_2) > 0.$$

Based on the assumptions of the model, θ_2 is unambiguously positive, as indicated, and the sign of θ_1 is indeterminate. However, as will be shown below, if decreasing absolute Arrow-Pratt risk aversion is assumed, θ_1 will be positive. Applying Cramer's rule to equation (13) obtains

$$(16) \quad \partial q_1^* / \partial c_2 = \frac{\begin{vmatrix} \theta_1 & 0 \\ \theta_2 & H_{22} \end{vmatrix}}{\overline{H}} = \frac{\begin{matrix} ? & - \\ \theta_1 & H_{22} \end{matrix}}{\overline{H}} = \begin{matrix} > \\ < \end{matrix}$$

+

and

$$(17) \quad \partial q_2^* / \partial c_2 = \frac{\begin{vmatrix} H_{11} & \theta_1 \\ 0 & \theta_2 \end{vmatrix}}{\overline{H}} = \frac{\begin{matrix} - & + \\ H_{11} & \theta_2 \end{matrix}}{\overline{H}} < 0.$$

+

In the two equations above, the + signs appearing above or below certain terms indicates terms that are unambiguously positive, just as the – signs appearing above other terms denote terms are that are unambiguously negative; however, the ? sign above a term indicates a term whose sign is indeterminate. This same convention in notation is also followed below in equations (20) and

(22). The economic interpretation of equation (17) is straightforward. Given the demand for a good is known with certainty, if its per unit cost increases, an expected utility-maximizing, multiproduct monopolist will produce less of it. Since the demand curve for q_2 is downward sloping, then $\partial p_2^*/\partial c_2 > 0$. If c_2 increases, then a utility-maximizing, multiproduct monopolist will decrease q_2^* and increase p_2^* .

As mentioned above, the sign of the expression in equation (16) cannot be determined unless additional restrictions are placed on the utility function. θ_1 will be positive if, like Sandmo (1971, p. 68), it is assumed that the utility function exhibits decreasing absolute Arrow-Pratt risk aversion. If $R_A(\pi)$ is the measure of absolute risk aversion, then

$R_A(\pi) = -U''(\pi)/U'(\pi)$, where it is assumed that $\partial R_A(\pi)/\partial \pi < 0$. Since

$U'(\pi) = -U''(\pi)/R_A(\pi)$, the first-order condition in equation (8) can be rearranged to obtain

$$(18) \quad 0 = \frac{z_1 U''(\pi_1)(a_0 - 2bq_1 - c_1)}{R_A(\pi_1)} + \frac{z_2 U''(\pi_2)(a_1 - 2bq_1 - c_1)}{R_A(\pi_2)} .$$

Since $R_A(\pi_2) < R_A(\pi_1)$, then

$$(19) \quad 0 < \frac{1}{R_A(\pi_2)} [z_1 U''(\pi_1)(a_0 - 2bq_1 - c_1) + z_2 U''(\pi_2)(a_1 - 2bq_1 - c_1)] = \frac{\theta_1}{R_A(\pi_2)} .$$

Given $R_A(\pi_2) > 0$, equation (19) ensures that θ_1 is positive. With the additional assumption of decreasing absolute Arrow-Pratt risk aversion, then equation (16) becomes

$$(20) \quad \frac{\partial q_1^*}{\partial c_2} = \frac{\theta_1 H_{22}}{H} < 0 .$$

According to equation (20), if the per unit cost of good 2 - - the good whose demand is known with certainty - - increases, the monopolist will produce less of good 1, the good with uncertain

demand. Since the demand curve for good 1 is downward sloping, a rise in c_2 , implies a fall in q_1^* , and an increase in p_1^* , or $\partial p_1^*/\partial c_2 > 0$. In the case of uncertain demand, the multiproduct monopolist will cross-subsidize - - it will increase the price of an unrelated good given an increase in the per unit cost of another good. Again this outcome is intuitive as it expands Sandmo's (1971) result. In terms of determining the optimal amount of good 1, the per unit cost of good 2 acts like fixed cost, and in the presence of uncertainty, an increase in fixed cost should lead to a decrease in q_1 .

To find the effect of a change in c_1 on the optimal quantities of q_1 and q_2 , the first-order conditions in equations (8) and (9) are once more evaluated at the solutions, $q_1^* = q_1^*(c_1, c_2)$ and $q_2^* = q_2^*(c_1, c_2)$, and differentiated with respect to c_1 . This comparative statics exercise results in the following two-equation system

$$(21) \quad \begin{bmatrix} H_{11} & 0 \\ 0 & H_{22} \end{bmatrix} \begin{bmatrix} \partial q_1^*/\partial c_1 \\ \partial q_2^*/\partial c_1 \end{bmatrix} = \begin{bmatrix} \theta_2 \\ 0 \end{bmatrix}.$$

Solving for $\partial q_1^*/\partial c_1$, equation (22) shows that even in the case of uncertain demand, if the unit cost of good 1 increases, the monopolist will produce less of q_1 , or

$$(22) \quad \partial q_1^*/\partial c_1 = \frac{\begin{vmatrix} \theta_2 & 0 \\ 0 & H_{22} \end{vmatrix}}{\overline{H}} = \frac{\theta_2 H_{22}}{\overline{H}} < 0.$$

Since the demand curve for good 1 is downward sloping, less q_1 means a higher p_1 or $\partial p_1^*/\partial c_1 > 0$. Conversely, if the demand for q_2 is known with certainty, then the firm's optimal level of q_2 is invariant to the value of c_1 or

$$(23) \quad \partial q_2^* / \partial c_1 = \frac{\begin{vmatrix} H_{11} & \theta_1 \\ 0 & 0 \end{vmatrix}}{\bar{H}} = 0 .$$

This result also coincides with intuition. If the demand for good 2 is known with certainty, why vary its output given a change in c_1 ? To do so would deviate from a known amount of output that maximizes profits in market 2, and would reduce expected utility by more than the initial increase in the cost of good 1. Given no change in q_2 , p_2^* remains constant, and $\partial p_2^* / \partial c_1 = 0$.

IV. Conclusions

This paper investigates how a change in per unit cost of one good affects the prices of a multiproduct monopolist. Using a simple model of a firm producing two goods with linear demand curves and constant per unit costs, two cases are investigated - - one where the demand for both the goods is known with certainty and the other where one of the goods has uncertain demand. Demand uncertainty is captured by assuming two possible states of the world where the demand curve for good 1 shifts parallel to the left or right. When the demand curves are known with certainty, changes in the per unit cost of one good does not affect the price of the other unrelated good. In this case, the profit-maximizing monopolist does not cross-subsidize increases in the cost of one good by charging a higher price on the other good. When the demand for good 1 is uncertain, an increase in the cost of good 2 will lead the expected utility-maximizing monopolist to charge higher prices for both good 1 and good 2. So in the case of uncertainty, a monopolist will cross-subsidize higher cost for one good with a higher price for the other unrelated good, demonstrating a key point of the paper. The next key result of the paper shows that an increase in the cost of the good with uncertain demand will result in less of it being produced, but this increase in cost has no effect on the output and price of the good whose demand is known with certainty.

This simple model opens several avenues for future research. Instead of a simple discrete probability approach to model uncertainty, the model should be expanded to continuous probability to verify that its results are robust. Other forms of demand uncertainty rather than parallel shifts in the demand curve need to be investigated to also ensure the robustness of the results. Additionally, other types of attitudes toward risk could be considered such as risk neutral or a risk lover. The model could be expanded to include economies of scope to get a fuller measure of how changes in cost affect prices of a multiproduct, imperfectly competitive firm. In this case, it may not be necessary to assume demand uncertainty in order to show cross-subsidization. Another extension of the model would be to disaggregate per unit cost into specific input prices and to examine how a change in input prices affect the prices charged by a multiproduct firm. These inputs could be common or unique to each product. Finally, the model may be expanded to show loss-leader behavior and examine the impact of changes in cost on such behavior. This future work would expand the use of the model and add to the literature of multiproduct firms.

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Figure 1

